AFOSR-TR- 81 -0868 LEVEL II

AD A108984

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Final Report

IMPLEMENTATION OF A POSTCRACKING MODEL FOR FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE

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September 1981

81 12 29 031

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 81 -0868	2. GOVT ACCESSION NO. AD-A1C8984	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) IMPLEMENTATION OF A POSTCRACKING MODEL FOR FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE		5. TYPE OF REPORT & PERIOD COVERED 1 Feb 81 - 31 Jul 81 FINAL
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) RAFAEL JIMENEZ-PEREZ		8. CONTRACT OR GRANT NUMBER(s) AFOSR 81-0092
9. PERFORMING ORGANIZATION NAME AND ADDRESS DEPARTMENT OF CIVIL ENGINEERING UNIVERSITY OF PUERTO RICO MAYAGUEZ, PUERTO RICO		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2307/D9
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BOLLING AIR FORCE BASE, DC 20332		12. REPORT DATE Sep 81
		13. NUMBER OF PAGES 64
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

REINFORCED CONCRETE
NONLINEAR RESPONSE
COMPUTER CODE
POSTCRACKING
BIAXIAL STRESSES

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A computer code has been developed to model the nonlinear response of reinforced concrete elements subjected to plane strain, plane stress or axisymmetric loading conditions. The material subroutine includes the following sources of nonlinear behavior: (a) Nonlinear stress strain curve for concrete as represented by the endochronic model; (b) stress-strain curve for the reinforcement including the elastic, plastic and strain hardening stages; (c) concrete anisotropy caused by complex stress states and cracking; and (d) the postcracking shear transfer mechanisms. The computer code has been checked

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UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

with a very limited amount of experimental data for plain concrete and for an initially cracked reinforced concrete panel subjected to biaxial stress states. In general, the code predicts results that agree satisfactorily with the experimental data but needs to the further the further checked against additional test cases.

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A comptuer code has been developed to model the nonlinear response of reinforced concrete elements subjected to plane strain, plane stress or axisymmetric loading conditions. The material subroutine includes the following sources of nonlinear behavior: (a) Nonlinear stress strain curve for concrete as represented by the endochronic model; (b) stress-strain curve for the reinforcement including the elastic, plastic and strain hardening stages; (c) concrete anisotropy caused by complex stress states and cracking; and (d) the postcracking shear transfer mechanisms.

The computer code has been checked with a very limited amount of experimental data for plain concrete and for an initially cracked reinforced concrete panel subjected to biaxial stress states. In general, the code predicts results that agree satisfactorily with the experimental data but needs to ~~the~~ be further checked against additional test cases.

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1. Introduction and Objectives:

1.1 Introduction:

The nonlinear analysis of reinforced concrete structures by the finite element method cannot be succesfully performed if the principal sources of material nonlinear behavior are not included in the formulation. The material characteristics that have to be considered are the nonlinear stress strain relation for the concrete, the stress strain relation for the reinforcement, concrete anisotropy due to complex stress states and cracking, postcracking shear transfer mechanisms at open cracks, and the concrete reinforcement bond slip relations.

This report presents a computer subroutine for the nonlinear analysis of reinforced concrete elements that includes the nonlinear stress strain relations for the concrete and the reinforcement, concrete anisotropy due to cracking and multiaxial stress states, and the postcracking shear transfer mechanisms present at a slightly open crack. The nonlinear behavior of the concrete is represented by the endochronic model (5) while the stress strain relation for the reinforcement represents the elastic, plastic, and strain hardening stages under monotonic or repeated loads. The postcracking shear transfer mechanisms included in the subroutine are the interface shear transfer mechanism on the rough surfaces of a cracked plane and the dowel action of the reinforcement crossing the crack. The subroutine developed uses the distributed crack approach to combine the stiffness matrix for the uncracked concrete with the stiffness relation for the cracks that predicts the incremental stresses induced in a reinforced concrete finite element by a set of prescribed incremental strains.

The different sections of this report describe first the constitutive relations used for the concrete, reinforcement and for the cracks, followed by a detailed discussion of all the code subroutines. Finally, several experimental tests are compared with the results predicted by the computer program to determine its validity.

1.2 Objectives:

The principal objective of this project was to develop a material subroutine that included the principal sources of nonlinear behavior in reinforced concrete. The specific objectives are:

A. Development of a material subroutine that calculates the incremental stress vector caused by a given vector of strains in a plane stress, strain or axisymetric finite element. The subroutine should consider the nonlinear behavior caused by the following sources:

1. Stress strain relation for concrete based on the endochronic model presented by Bazant (2 , 3).

2. Concrete anisotropy caused by cracking and multiaxial stress states.

3. Postcracking shear transfer mechanisms. Both the interface shear transfer and the dowel action stiffness representation are included in the subroutine.

4. Stress-strain relation for reinforcement that models the elastic, plastic, and strain hardening stages for monotonic and repeated loads.

B. Comparison of the material subroutine code predictions with available experimental data to establish the valitidity of the proposed formulation.

2. Constitutive Relations for Nonlinear Analysis of Reinforced Concrete

2.1 Introduction:

The following sections present the theoretical background required to model the nonlinear behavior of reinforced concrete in the computer program. First, the constitutive relations used for concrete, as given by the endochronic model, are discussed, followed by the constitutive relations employed to model the behavior of the reinforcement subjected to monotonic or repeated loads. For elements that have not cracked, the concrete and steel stiffness relations in terms of the global coordinates can be added to obtain an incremental stress-strain relation for the reinforced concrete element. For elements that have cracked, however, the incremental stress-strain relation has to consider the constitutive relation for the cracks, presented on the last section of this chapter.

2.2 Constitutive Relations for Concrete:

Several theories have been devloped to predict the response of plain concrete to multiaxial stress states among which are the linear and nonlinear elasticity theories, the work hardening plasticity theories, the plastic fracturing theory, and the endochronic theory (4,8). Of these theories, the endochronic theory has received particular attention as it provides a continous model for the nonlinear representation of concrete without the explicit formulation of a yield condition and hardening rules. The endochronic model developed by Bazant and co-workers (2 , 5) have been used succesfully to predict the nonlinear stress-strain curve for concrete subjected to monotonic or repeated loading.

The endochronic theory for concrete initially proposed by Bazant (2) introduced a non decreasing scalar variable, denominated intrinsic time, to represent the accumulation of inelastic strains as a function of the strain increments applied to the element. The intrinsitive increments were assumed to be sensitive to the hydrostatic pressure. The theory also modelled the strain hardening and strain softening regions of the stress strain curve for concrete, the inelastic dilatancy due to shear straining measured by another non-decreasing scalar variable, and the dependance of the incremental elastic moduli on the dilatancy measure.

The initial endochronic model proposed by Bazant was later refined (5), to include the additional inelastic strains caused by hydrostatic compression, volume changes in the strain softening range of the stress strain curve, dependance of material parameters on strength, and an improved description of the strain softening behavior under monotic or repeated loading. The refined endochronic model was used to represent the nonlinear behavior of concrete in the material subroutine.

2.2.1 Endochronic Model for Trialxial Behavior of Concrete

The stress strain relations for the endochronic model are given in terms of the deviatoric and volumetric relations, as follows:

$$\Delta e_{ij} = \frac{\Delta S_{ij}}{2G} + \Delta e_{ij}'' \qquad \text{1a}$$

$$\Delta \varepsilon = \frac{\Delta \sigma}{3K} + \Delta \varepsilon'' \qquad \text{1b}$$

Where: Δe_{ij}, ΔS_{ij} = deviatoric components of strain and stress tensor, respectively.

$\Delta\varepsilon$, $\Delta\sigma$ = volumetric component of strain and stress tensor, respectively.

$\Delta e_{ij}''$ = inelastic deviator strain increment.

$\Delta\varepsilon''$ = inelastic volumetric strain increment

K, G = Bulk and shear modulus.

i,j = Cartesian coordinates indexes.

The volumetric components of the strain and stress vectors are computed from:

$$\Delta\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \qquad \text{2a}$$

$$\Delta\sigma = \sigma_{11} + \sigma_{22} + \sigma_{33} \qquad \text{2b}$$

while the deviatoric components are obtained from:

$$\Delta e_{ij} = \Delta \epsilon_{ij} - \delta_{ij} \frac{\Delta \epsilon}{3} \quad 3a$$

$$\Delta S_{ij} = \Delta \sigma_{ij} - \delta_{ij} \frac{\Delta \sigma}{3} \quad 3b$$

Where: δ_{ij} = Kronecer Delta given by:

= 1 for i = j

= 0 for i $\neq$ j

The inelastic deviatior strain increment is a function of the distortion intrinsic time parameter, Z, and of the deviatonic stress, S_{ij}, given by:

$$\Delta e''_{ij} = \frac{S_{ij}}{2G} Z \quad 4$$

The inelastic volumetric strain increment is a function of the volumetric stress, σ, the inelastic dilatancy λ, the shear compaction λ', and of the compaction intrinsic time parameter Z'. The compaction intrinsic time parameter Z' have been introduced to account for the volumetric inelastic strains caused by hydrostatic stress states, while the shear compaction parameter λ' accounts for the increased volumetric strains observed in triaxial stress tests when compared to hydrostatic stress tests. The inelastic volumetric strain is given by:

$$\Delta \epsilon'' = \Delta\lambda + \frac{\sigma}{3K} \Delta Z' + \Delta\lambda' \quad 5$$

Thus, the inelastic deviatoric and volumetric strains are a function of the distortion intrinsic time Z, the compaction intrinsic time Z', the inelastic dilatancy λ, shear compaction λ', the bulk and shear modulus, and the volumetric and deviatoric stress components present in the element. The endochronic parameters and the bulk and shear modulus are computed from the set of equations summarized in Appendix A1. It should be noted that the functions used to calculate the intrinsic time parameters, inelastic dilatancy and shear compaction are a function of the current stress and strain invariants, and of the principal stresses in the element. Therefore, for a prescribed strain increment, the associatted stress increment has to be computed in an iterative fashion until the endochronic parameters at any given iteration converge to those of the previous iteration. In the computer code, the iteration is terminated when the difference between the previous and current values of the inelastic dilatancy and shear compaction parameters in within 0.01.

It should be noted that the accumulated values of several endochronic parameters, as well as for the stress and strain vectors, are required for the equations given in Appendix A1. The current value of any parameter at the end of a strain increment is calculated from the increment of said parameter computed when the iteration is finished, and the value obtained in the previous strain increment.

The stress strain relations in terms of deviatoric and volumetric components given by Equation 1 can be combined to obtain an incremental stress strain relation in terms of the element coordinates, using the relations given by Equation 3. Rearranging Equations 1, we have

$$\Delta S_{ij} = 2G\, \Delta e_{ij} - 2G\, \Delta e''_{ij} \qquad \text{6a}$$

$$\Delta \sigma = 3K\, \Delta \varepsilon - 3K\, \Delta \varepsilon'' \qquad \text{6b}$$

If we define the second term of the right hand side of Equation 6 as an equivalent inelastic deviatoric and volumetric stress increment, we then can rearrange Equation 6 in the following form:

$$\Delta S_{ij} + \Delta S''_{ij} = 2G\, \Delta e_{ij} \qquad \text{7a}$$

$$\Delta \sigma + \Delta \sigma'' = 3K\, \Delta \varepsilon \qquad \text{7b}$$

Where:

$\Delta S''_{ij}$ = deviatoric stress increment

$\Delta \sigma''$ = volumetric inelastic stress increment

The incremental stress-strain relation in terms of the element coordinates can then he obtained by adding Equations 7a and 7b. Thus

$$\Delta \sigma_{ij} + (\Delta S''_{ij} + \delta_{ij} \Delta \sigma'') = 2G\, \Delta e_{ij} + 3K\, \delta_{ij}\, \Delta \varepsilon \qquad 8$$

Where:

$\Delta\sigma_{ij}$ elastic stress increment referred to the element coordinates

In matrix form, the incremental stress strain relation is then given by the following relation, where the axis directions are defined in Figure 1.

$$\begin{Bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{22} \\ \Delta\sigma_{33} \\ \Delta\sigma_{12} \\ \Delta\sigma_{13} \\ \Delta\sigma_{23} \end{Bmatrix} + \begin{Bmatrix} \Delta\sigma''_{11} \\ \Delta\sigma''_{22} \\ \Delta\sigma''_{33} \\ \Delta\sigma''_{12} \\ \Delta\sigma''_{13} \\ \Delta\sigma''_{23} \end{Bmatrix} = \begin{bmatrix} D_1 & D_2 & D_2 & 0 & 0 & 0 \\ D_2 & D_1 & D_2 & 0 & 0 & 0 \\ D_2 & D_2 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_3 \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{22} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{12} \\ \Delta\varepsilon_{13} \\ \Delta\varepsilon_{23} \end{Bmatrix} \qquad 9a$$

Where:

$$D_1 = K + \frac{4}{3} G \qquad 9b$$

$$D_2 = K - \frac{2}{3} G \qquad 9c$$

$$D_3 = 2G \qquad 9d$$

For plane stress conditions, the following boundary conditions are known:

$$\Delta\sigma_{22} = \Delta\sigma_{13} = \Delta\sigma_{23} = \Delta\varepsilon_{12} = \Delta\varepsilon_{23} = 0 \qquad 10$$

Hence the incremental stress-strain relation is given by:

$$\begin{Bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{33} \\ \Delta\sigma_{13} \end{Bmatrix} + \begin{Bmatrix} \Delta\sigma^{p}_{11} \\ \Delta\sigma^{p}_{33} \\ \Delta\sigma''_{13} \end{Bmatrix} = \begin{bmatrix} D_1-D_2^2/D_1 & D_2-D_2^2/D_1 & 0 \\ D_2-D_2^2/D_1 & D_1-D_2^2/D_1 & 0 \\ 0 & 0 & D_3 \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{13} \end{Bmatrix} \qquad 11a$$

Where:

$$\Delta\sigma^{p}_{11} = \Delta\sigma''_{11} - \frac{D_2}{D_1} \Delta\sigma''_{22} \qquad 11b$$

$$\Delta\sigma^{p}_{33} = \Delta\sigma''_{33} - \frac{D_2}{D_1} \Delta\sigma''_{22} \qquad 11c$$

and the strain component in the normal direction is given by:

$$\Delta\varepsilon_{22} = \frac{\Delta\sigma''_{22} - D_2 (\Delta\varepsilon_{11} + \Delta\varepsilon_{33})}{D_1} \qquad 11d$$

For plain strain conditions, the following boundary conditions are known:

$$\Delta\sigma_{12} = \Delta\sigma_{23} = \Delta\varepsilon_{22} = \Delta\varepsilon_{12} = \Delta\varepsilon_{23} = 0 \qquad 12$$

Hence, the following incremental stress-strain relation results:

$$\begin{Bmatrix} \Delta\sigma_{11} \\ \Delta\sigma_{33} \\ \Delta\sigma_{13} \end{Bmatrix} + \begin{Bmatrix} \Delta\sigma''_{11} \\ \Delta\sigma''_{33} \\ \Delta\sigma''_{13} \end{Bmatrix} = \begin{bmatrix} D_1 & D_2 & 0 \\ D_2 & D_1 & 0 \\ 0 & 0 & D_3 \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{11} \\ \Delta\varepsilon_{33} \\ \Delta\varepsilon_{13} \end{Bmatrix} \qquad 13$$

The normal stress in the third direction can be computed from:

$$\Delta\sigma_{22} = D_2 \, (\Delta\varepsilon_{11} + \Delta\varepsilon_{33}) - \Delta\sigma''_{22} \qquad 14$$

Equations 11, 13, and 14 have been implemented in the computer code to calculate the stress increments for plane strain or plane stress conditions. For a prescribed vector of incremental strains, the corresponding elastic incremental stress vector can be computed from these equations once the elastic stiffness coefficients and the inelastic stress increment vector have been calculated from the endochronic parameters.

2.2.2 Linearization of Endochronic Formulation

The incremental stress-strain relation given by Equation 9a is expressed in terms of an elastic and an inelastic stress vector. This relation is adequate when the concrete stiffness matrix does not have to be combined with the crack constitutive relation. For this cases, the endochronic stress strain relation needs to be formulated in the following form:

$$\{\Delta\sigma\} = [D'] \; \{\Delta\varepsilon\} \qquad 15$$

Where:

$\{\Delta\sigma\}$ = incremental stress vector referred to the element coordinates.

$\{\Delta\varepsilon\}$ = prescribed incremental strain vector referred to the element coordinates.

$[D']$ = matrix of elastic stiffness coefficient for linearized endochronic formulation.

The matrix of elastic stiffness coefficients can be calculated if the curved inelastic stiffness locus of the endochronic theory is replaced by a tangent to the curved locus at the point of assumed strain increment (1). This requirement can be established if the equation used to compute the distortion measure parameter, (Equation A1.4 in Appendix A1) is replaced by the following relation.

$$\Delta\xi = \frac{\Delta e_{ij}}{2\sqrt{J_2(\Delta e_{ij})}} \cdot \Delta e_{ij} \qquad 16$$

The deviatoric inelastic stress vector component can then be expressed by:

$$\Delta S''_{ij} = S_{ij} \quad \frac{F}{Z_1 f} \quad (B_{ij}) \quad \Delta e_{ij} \qquad 17a$$

Where:

$$B_{ij} = \frac{\Delta e_{ij}}{2\sqrt{J_2(\Delta e_{ij})}} \qquad 17b$$

The volumetric inelastic stress vector component can then be expressed by:

$$\Delta\sigma = \frac{3\sigma H}{Z_2 h} \Delta\varepsilon + 3K(\ell \cdot L + \ell' \cdot L') B_{ij} \Delta e_{ij} \qquad 18$$

Hence, the incremental stress strain relations in deviatoric and volumetric components are given by:

$$\Delta S_{ij} = 2G \Delta e_{ij} - S_{ij} \left(\frac{F \quad B_{rs}}{Z_1 f} \right) \cdot \Delta e_{rs} \qquad 19a$$

$$\Delta\sigma = 3K \Delta\varepsilon - \left(\frac{\sigma H}{Z_2 h} \right) \Delta\varepsilon + 3K \ (\ell : L + \ell' : L') \ B_{rs} \cdot \Delta e_{rs} \qquad 19b$$

The total incremental stress vector, referred to the element coordinates is then given by the following indicial relation:

$$\Delta\sigma_{ij} = (2G\delta_{ir}\delta_{js} - \frac{S_{ij}F}{Z_1 f} B_{rs} - 3K \ (\ell.L + \ell'.L') \ \delta_{ij} B_{rs}) \ \Delta e_{rs}$$

$$+ \left(K - \frac{\sigma H}{Z_2 h} \right) \delta_{ij} \ \delta_{km} \ \Delta\varepsilon_{km} \qquad 20$$

The elastic stiffness coefficients in Equation 15 are then given by the following equation:

$$D_{ijkm} = 2G\,\delta_{ik}\,\delta_{jm} + \left(K - \frac{2}{3}G - \frac{\sigma H}{Z_2 h}\right)\delta_{ij}\,\delta_{km}$$

$$- \left(\frac{S_{ij}}{Z_1 f}F + 3K\,(\ell \cdot L + \ell' \cdot L')\,\delta_{ij}\right)\left(B_{km} - \frac{B_{nn}}{3}\delta_{km}\right) \qquad 21$$

In the computer program, the above calculations are performed only for the in plane normal and tangential stresses and strains, and for the normal stress and strain in the direction perpendicular to the plane considered. Thus, once the deviatoric stress and strain components for each direction is calculated the coeficients B_{ij} are computed together with its volumetric component.

Let the variable x be defined by:

$$X_{ij} = \frac{S_{ij}}{Z_1 f}F + 3K\,(\ell \cdot L + \ell' \cdot L') \qquad 22$$

Then, the elements in matrix $[D']$ are given by the following equations for the general case of plane stress or strain:

$$D'(1,1) = K + \frac{4}{3}G - \frac{\sigma H}{3Z_2 h} - X_{11}\left(B_{11} - \frac{B_{nn}}{3}\right) \qquad 23a$$

$$D'(2,2) = K + \frac{4}{3}G - \frac{\sigma H}{3Z_2 h} - X_{22}\left(B_{22} - \frac{B_{nn}}{3}\right) \qquad 23b$$

$$D'(3,3) = K + \frac{4}{3}G - \frac{\sigma H}{3Z_2 h} - X_{33}\left(B_{33} - \frac{B_{nn}}{3}\right) \qquad 23c$$

$$D'(4,4) = 2G - X_{44}\,B_{44} \qquad 23d$$

$$D'(1,2) = K - \frac{2}{3}G - \frac{\sigma H}{3Z_2 h} - X_{11}\left(B_{22} - \frac{B_{nn}}{3}\right) \qquad 23e$$

$$D'(1,3) = K - \frac{2}{3}G - \frac{\sigma H}{3Z_2 h} - X_{11}\left(B_{33} - \frac{B_{nn}}{3}\right) \qquad 23f$$

$$D'(1,4) = -X_{11}\,B_{44} \qquad 23g$$

$$D'(2,1) = K - \frac{2G}{3} - \frac{\sigma H}{3Z_2 h} - X_{22}\left(B_{11} - \frac{B_{nn}}{3}\right) \qquad 23h$$

$$D'(2,3) = K - \frac{2G}{3} - \frac{\sigma H}{3Z_2 h} - X_{22}\left(B_{33} - \frac{B_{nn}}{3}\right) \qquad 23i$$

$$D'(2,4) = -X_{22}\,B_{44} \qquad 23j$$

$$D'(3,1) = K - \frac{2G}{3} - \frac{\sigma H}{3Z_2 h} - X_{33}\left(B_{11} - \frac{B_{nn}}{3}\right) \qquad 23K$$

$$D'(3,3) \quad K \quad \frac{2G}{3} \quad [illegible] \quad X_{33}\left(B_{33} - \frac{B_{nn}}{3}\right) \qquad 23l$$

$$D'(3,4) = -X_{33}\, B_{44} \qquad 23m$$

$$D'(4,1) = -X_{44}\left(B_{11} - \frac{B_{nn}}{3}\right) \qquad 23n$$

$$D'(4,2) = -X_{44}\left(B_{22} - \frac{B_{nn}}{3}\right) \qquad 23o$$

$$D'(4,3) = -X_{44}\left(B_{33} - \frac{B_{nn}}{3}\right) \qquad 23p$$

Where:

$$B_{nn} = B_{11} + B_{22} + B_{33} \qquad 23q$$

The stress strain relation given in Equation 15 is referred to the element local coordinate system, which for the computer program has been assumed to be oriented along the principal stress axis. To obtain the incremental stress strain relation in terms of global coordinates the stiffness matrix $[D']$ shall be transformed to the global axis my means of the following relation:

$$[D']_g = [T]^T \, [D'] \, [T] \qquad 24$$

Where:

$[D']_g$ = stiffness matrix for concrete in global coordinates

$[T]$ = transformation matrix given by:

$$[T] = \begin{bmatrix} C^2 & S^2 & 0 & 2CS \\ 0 & 1 & 0 & 0 \\ S^2 & C^2 & 0 & -2CS \\ -CS & CS & 0 & C^2-S^2 \end{bmatrix} \qquad 25$$

C = $\cos\alpha$

S = $\sin\alpha$

α = angle between global and local coordinate axis

(See Figure 1)

2.3 Constitutive Relations for Reinforcement:

The constitutive relations for the reinforcement subjected to monotonic or repeated loading are required to model adequately the nonlinear behavior of reinforced concrete structures. A typical stress-strain curve for the reinforcement subjected to monotonic loading is shown in Figure 2. Three different stages of behavior are evident, namely, the elastic range, the plastic range, and the strain hardening range. During the elastic stage, the relation between stress and strain is linear and is given by the modulus of elasticity of the reinforcement. For the plastic range, the strain increases continously at a constant stress and the modulus of elasticity is zero. For the strain hardening region, a nonlinear relation exists between stress and strain, and a much more complicated stiffness relation has to be determined from experimental data.

The following relations have been suggested (7) to model the stress-strain curve for monotonic loading up to failure.

Elastic Region ($\varepsilon<\varepsilon_y$): $f_s = E\varepsilon$ (26a)

$E_s = 29000$ Ksi (26b)

Plastic region ($\varepsilon_y<\varepsilon<\varepsilon_{sh}$): $f_s = f_y$ (26c)

$E = 0$ (26d)

Strain hardening region ($\varepsilon_{sh}<\varepsilon<\varepsilon_{su}$):

$$f_s = f_y \left[\frac{112(\varepsilon-\varepsilon_{sh})+2}{60(\varepsilon-\varepsilon_{sh})+2} + \frac{(\varepsilon-\varepsilon_{sh})}{(\varepsilon_{su}-\varepsilon_{sh})}\left(\frac{f_{su}}{f_y} - 1.7\right)\right] \qquad \text{26e}$$

$$E = \frac{104 f_y}{(60(\varepsilon-\varepsilon_{sh})+2)^2} + \frac{f_{su} + 1.7 f_y}{f_y(\varepsilon_{su} - \varepsilon_{sh})} \qquad \text{26f}$$

Where:

f_s = steel stress at strain ε

f_y = steel yield stress

f_{su} = steel ultimate stress

ε = actual steel strain

ε_y = Steel yield strain

ε_{sh} = steel strain at the initiation of strain hardening

ε_{su} = Steel ultimate strain

E_s = Elastic modulus of elasticity

E = Modulus of elasticity at given strain

The monotonic stress-strain curve serves as an envelope for specimens subjected to repeated loading. Upon initial loading, the stress-strain is similar to that for monotonic loading. Upon unloading, the stiffness is similar to the linear loading stiffness but a residual displacement will be observed if the specimen has been strained to the plastic range. When the specimen is subsequently loaded, the stress-strain relation is linear until it coincides with the monotonic stress-strain curve, whereupon it follows the virgin stress-strain relationship.

The above constitutive relations are valid for uniaxial stress states only. For reinforcement oriented along 3 arbitrary directions, the constitutive relation is given by:

$$\left\{\Delta\sigma\right\}^s = \left[D\right]^s \left\{\Delta\varepsilon\right\} \tag{27}$$

Where:

$\left\{\Delta\sigma\right\}^s$ = incremental stress in reinforcement along bar orientations

$\left\{\Delta\varepsilon\right\}$ = prescribed incremental strain in reinforcement along bar orientations

$\left[D\right]^s$ = reinforcement stiffness matrix

The reinforcement stiffness matrix referred to the bar directions as given by:

$$\left[D\right]^s = \begin{bmatrix} \rho_1 E_1 & 0 & 0 \\ 0 & \rho_2 E_2 & 0 \\ 0 & 0 & \rho_3 E_3 \end{bmatrix}$$

Where:

ρ_1, ρ_2, ρ_3 = reinforcement ratios along bar directions 1, 2, and 3.

$E_1\ E_2, E_3$ = modulus of elasticity of reinforcement along bar directions 1, 2, and 3.

In general, the bar directions do not coincide with the principal stress orientation used to compute the concrete stiffness matrix. In this case, the stiffness matrix for the reinforcement must be transformed to the principal axis by the following relation:

$$[D]_p^s = [T]^T \; [D]^s \; [T] \tag{29}$$

Where:

$[D]_p^s$ = stiffness matrix for the reinforcement referred to the principal axis.

The incremental stresses in the reinforcement can be calculated once the prescribed incremental strains in the reinforcement and the modulus of elasticity for each bar direction has been determined. The modulus of elasticity for each bar direction required by Equation 28 is obtained from the stress-strain curve for the reinforcement given in Equation 26. It must be noted however, that when the strain in the reinforcement is larger than the yield strain ϵ_y and the element is unloaded, a new yield point must be defined at the maximum strain achieved during the loading step. The computer subroutine that calculates the incremental steel stresses determines whether the bars are being loaded or unloaded and computes the stiffness for each bar direction according to the prescribed total strains. For bar directions that are unloaded the code automatically shifts the position of the yield strain to obtain the correct stiffness for the reinforcement direction considered.

2.4 Constitutive Relations for Postcracking Shear Transfer Mechanisms:

The postcracking shear transfer mechanisms have been identified as the interface shear transfer on the rough surfaces of the crack and the dowel action of the reinforcement crossing the crack. The interface shear transfer mechanism is used to describe both the bearing and frictional forces generated at open cracks as the protuding particles on each side of the cracked surface come into contract. Various experimental investigations (9 ,11 ,15) have indicated that the major physical parameters that affect the interface shear transfer stiffness are the initial crack width, the axial stiffness of the reinforcement crossing the crack, and the application of cyclic loading.

The dowel action mechanism is provided primarily by the bending and shearing stiffness of the reinforcement as a tangential displacement is experienced along the crack length. The dowel stiffness of the bar depends mainly on the bar diameter, the concrete tensile strength, the axial stress in the reinforcement and the application of cyclic loadings.

On an cracked surface of a reinforced concrete element, both mechanisms are activated simultaneously to transfer the applied shear force across the crack. A complete mathematical description of the forces and displacements experienced across the crack can be obtained if a flexibility relationship of the following form can be established:

$$\begin{Bmatrix} \Delta\delta_n \\ \Delta\delta_s \end{Bmatrix} = A_c \begin{bmatrix} F_1 & F_2 \\ F_3 & F_4 \end{bmatrix} \begin{Bmatrix} \Delta\sigma_n \\ \Delta\sigma_{nt} \end{Bmatrix} \qquad 30$$

Where:

$\Delta\delta_n$ = normal displacement at crack

$\Delta\delta_s$ = tangential displacement at crack

$\Delta\sigma_n$ = normal stress at crack

$\Delta\sigma_{nt}$ = tangential stress at crack

F_1, F_2, F_3, F_4 = flexibility coefficients at crack

A_c = area of shear plane

The flexibility coefficients required for Equation 30 have been derived in Ref. 12 by applying incremental unit normal and shearing stress as at the cracked surface and calculating the associatted incremental normal and shear displacements.

Coefficient F_1 reflects the change in normal displacement experienced at the crack when an incremental unit normal force is applied to the shear plane. This coefficient can be simply described by the inverse of the normal restraint stiffness provided by the reinforcement crossing the crack, defined by:

$$K_n = \frac{\Delta P}{\Delta \delta_n} \qquad 31$$

Hence, coefficient F_4 is given by:

$$F_4 = \frac{1}{K_n} = \frac{1}{8240 d_b^{2}} \qquad 32$$

Where the normal restraint stiffness K_n, is calculated from the relation proposed by Jiménez, et al. (13)

The flexibility coefficient F_2 represents the increase in normal displacement caused by the applied shear. If it is assumed that the increase in crack width or normal displacement is caused mainly by the interface shear transfer stresses, the increase in crack width can be calculated from the normal stresses induced by the applied shear. Based on an expression proposed by Reinhardt and Walraven (17), the change in crack width can be calculated from:

$$F_2 = \frac{1}{K_n} \left[\frac{0.176c^{-0.63} + (0.22c^{-.552} - 1.034)\, f'_c}{0.1353c^{-0.8} + (0.164c^{-.707} - 1.379)\, f'_c} \right] \alpha \qquad 33$$

Where:

α = ratio of interface shear transfer stiffness to the sum of the interface shear transfer and dowel action stiffness.

f'_c = concrete compressive strength (ksi).

c = initial crack width (in.).

The flexibility coefficient F_3 can be calculated if it is assumed that the increase in shear displacement is caused by the reduction in the interface shear transfer stiffness associated with a larger crack width. In mathematical terms,

$$F_3 = \frac{d\Delta\delta_s}{d\Delta\sigma_n} = \frac{d\Delta\delta_s}{d\Delta\delta_c} \times \frac{d\Delta\delta_c}{d\Delta\sigma_n} \qquad 34$$

The first derivative of the change in shear displacement with respect to the change in crack width will be obtained once the equation for the coefficient F_4 is obtained. Note that in Equation 34, the change in normal displacement with respect to the change in normal stress is proportional to the normal restraint stiffness. Thus, Equation 34 can be rewritten as:

$$F_3 = \frac{1}{K_n} \cdot \frac{d\Delta\delta_s}{d\Delta\delta_c} \qquad 35$$

The flexibility coefficient F_4 represents the incremental shear displacement experienced at the crack when an incremental unit tangential shear force is transferred across the crack. The shear displacement is inversely proportional to the total stiffness provided by the interface shear transfer and the dowel action mechanisms. Given the stiffness of both mechanisms, the function F_4 can be calculated from the following equation:

$$F_1 = \frac{1}{K_a + K_d} \qquad 36$$

Where:

K_a = interface shear transfer stiffness (K/in)

K_d = dowel stiffness of reinforcement crossing the crack (k/in)

Based on a review of several relations available for the interface shear transfer and dowel action stiffness, the following equation was selected from Reference 12.

<u>Interface Shear Transfer Stiffness:</u>

$$K_a = \frac{A_c}{3.9(c-0.002) + 1.09\text{x}10^{-7}\left(3.4 \times 10^5 - \frac{K_n}{c}\right)} \qquad 37$$

Where:

K_a = interface shear transfer stiffness

K_n = normal restraint stiffness

Dowel Action Stiffness:

$$K_d = B_2 + \frac{\left[\left[\frac{\alpha_2}{\alpha_1} V_{du} - \frac{B_2}{0.5} \delta_d\right] \frac{B_2}{0.5} + \frac{2B_1 d_b \sqrt{f'_c}\, V_{du}}{3 \times 10^{-3} \alpha_1}\right]}{2\left[\left[\frac{\alpha_2}{\alpha_1} V_{du} - \frac{B_2}{0.5} \delta_d\right]^2 + 4\, \frac{B_1 d_b \sqrt{f'_c}\, V_{du}\, \delta_d}{3 \times 10^{-3} \alpha_1}\right]^{1/2}} \qquad 38$$

Where:

$$B_1 = (1 - 2f_s/f_y) \geq 0 \qquad 39$$

$$B_2 = \frac{f_s d_b \sqrt{f'_c}}{3\text{x}10^{-3} \alpha_1 f_y} \qquad 40$$

and

α_1 = 2 for $V_d < 0.9\ V_{du}$ 41

α_1 = 62 for $V_d > 0.9\ V_{du}$

α_2 = 0 for $V_d < 0.9\ V_{du}$ 42

α_2 = −54 for $V_d > 0.9\ V_{du}$

δ_d = dowel displacement (in)

f_s = axial stress in reinforcement (Ksi)

f_y = yield stress of reinforcement (ksi)

V_{du} = ultimate dowel capacity of reinforcement (K)

d_b = bar diameter (in)

The ultimate dowel capacity of the reinforcement is controlled by whether the dowel will fail by yielding of the reinforcement or by concrete splitting. Thus, the ultimate dowel load is given by the smaller of the values predicted by the following relations:

Failure by yielding of the reinforcement:

$$V_{dy} = 0.92 d_b^2 \sqrt{f_y\ f'_c} \qquad 43$$

Where:

V_{dy} = dowel failure load caused by yielding of the reinforcement (Kips)

f_y - yield stress of the reinforcement (Ksi)

<u>Failure by concrete splitting:</u>

$$V_{do} = \frac{d_b b_n}{n_b} \left[0.47 + \frac{0.54 c_m}{\frac{b_n + d_b}{n_b^2}} \right] \qquad 44$$

Where:

V_{do} = dowel failure load caused by concrete splitting (Kips)

b_n = net width of section perpendicular to load direction (in).

n_b = number of bars per layer.

c_m = smaller of side or bottom concrete cover of the reinforcement (in)

Thus, the flexibility coefficient F_4 can be obtained from Equation 36 once the interface shear transfer and dowel action stiffness have been calculated from Equations 37 and 38.

The equation for coefficient F_3 can now be presented once the first derivative of Equation 37 with respect to the initial crack width is computed:

$$F_3 = \frac{1}{K_n} \left[3.9 - \frac{1.09\text{x}10^{-7} \quad K_n}{c^2} \right] \Delta V_o \qquad 45$$

Where:

ΔV_o = shear stress increment applied in previous step.

2.5 <u>Constitutive Relations for Cracked Reinforced Concrete</u>

The incremental stress vector induced by a prescribed strain increment in a reinforced concrete element can be obtained from the incremental stress vectors sustained separately by the concrete and the reinforcement, provided that the concrete element has not cracked. If we assume that the average strains in the concrete and the reinforcement are equal, then the total incremental stress can be calculated from:

$$\{\Delta\sigma\}^T = \left[[D]^c + [D]^s \right] \{\Delta\varepsilon\} \qquad 46$$

Where:

$\{\Delta\sigma\}^T$ = total incremental stress vector

$\{\Delta\varepsilon\}$ = prescribed incremental strain vector

$[D']$ = stiffness matrix of uncracked concrete element

$[D]^S$ = stiffness matrix of reinforcement

For reinforced concrete elements where the principal tensile stress has exceeded the maximum tensile strength of the concrete, the incremental stress vector is a function of the tangential and normal stresses transferred across the crack. For this cases the constitutive relation given in Equation 46 has to be modified as described subsequently.

When the principal tensile stress exceeds the maximum tensile strength of the concrete the prescribed incremental strain calculated for the current step has to be divided into the incremental strain required for the element to crack and the remaining incremental strain necessary to complete the total incremental strain computed for the current step. Hence,

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon\}^1 + \{\Delta\varepsilon\}^2 \qquad 47$$

Where:

$\{\Delta\varepsilon\}$ = total incremental strain at current time step

$\{\Delta\varepsilon\}^1$ = incremental strain required for crack initiation

$\{\Delta\varepsilon\}^2$ = incremental strain required to complete the total strain increment assigned to current step.

The incremental strain required for the element to crack is estimated from the proportion of the stress increment at which the principal stress equals the tensile strength of the concrete. Said proportion is given by:

$$P = \frac{f_t - \sigma_{pl}^{n-1}}{\sigma_{pl}^{n} - \sigma_{pl}^{n-1}} \qquad 48$$

Where:

P = Proportion of stress increment at which cracking ocurred

f_t = tensile strength of concrete

σ_{p1}^{n} = principal tensile stress at current step.

σ_{p1}^{n-1} = principal tensile stress at previous step

Thus, the incremental strain at which cracking ocurred is given by:

$$\left\{\Delta\varepsilon\right\}^1 = P\left\{\Delta\varepsilon\right\} \qquad 49$$

The remaining incremental strain to be applied to the cracked element during the current step is then given by:

$$\left\{\Delta\varepsilon\right\}^2 = (1 - P)\left\{\Delta\varepsilon\right\} \qquad 50$$

The incremental strain applied to the cracked element required to complete the current step is distributed between the cracked and uncracked sections of the element according to the following equations:

$$\left\{\Delta\varepsilon\right\}^{cr} = \alpha\left\{\Delta\varepsilon\right\}^2$$

$$\left\{\Delta\varepsilon\right\}^{unc} = (1-\alpha)\left\{\Delta\varepsilon\right\}^2$$

Where:

$\left\{\Delta\varepsilon\right\}^{cr}$ = strains contributed by the cracks in the element

$\left\{\Delta\varepsilon\right\}^{unc}$ = strain contributed by the uncracked section of concrete within the cracks.

α = proportion of incremental strain provided by cracks within the element

Once the strains contributed by the cracks are known the average normal and tangential displacements can be determined from the crack spacing, as given by the following relations:

$$\begin{Bmatrix} \Delta\delta_n \\ \Delta\delta_s \end{Bmatrix} = S_c \begin{Bmatrix} \Delta\varepsilon_n \\ \Delta\varepsilon_{nt} \end{Bmatrix}^{cr} \qquad 52$$

Where:

$\Delta\varepsilon_n^{cr}$ = incremental strain normal to crack contributed by the cracks

$\Delta\varepsilon_t^{cr}$ = incremental strain parallel to crack contributed by the cracks

S_c = crack spacing

It should be noted that in terms of the four stress components considered in the computer code, the constitutive relation for the cracks presented in Equation 30 can now be expressed in terms of the crack strains by the following relation:

$$\begin{Bmatrix} \Delta\varepsilon_n \\ 0 \\ 0 \\ \Delta\varepsilon_{nt} \end{Bmatrix}^{cr} = \begin{bmatrix} F_1 & 0 & 0 & F_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ F_3 & 0 & 0 & F_4 \end{bmatrix} \begin{Bmatrix} \Delta\sigma_n \\ 0 \\ 0 \\ \Delta\sigma_{nt} \end{Bmatrix}^{cr} \qquad 52$$

The strains contributed by the uncracked concrete between the cracks are used to determine the endochronic parameters required to calculate the concrete stiffness matrix given in Equation 15. As the stresses in the uncracked concrete and in the crack have to be similar, the proportion of incremental strain taken by the uncracked concrete (See Equation 51) is determined in an iterative fashion from the following relation once the crack flexibility matrix and the concrete stiffness matrix are known:

$$\{\Delta\varepsilon\}^{un} = \left[\frac{A_c}{S_c} [F][D] + [I] \right]^{-1} \{\Delta\varepsilon\} \qquad 53$$

If the new value for the uncracked strains are within tolerable limits of the assumed uncracked strains, then the convergence requirement that the stresses in the solid concrete be equal to the stresses in the cracked concrete has been satisfied. Otherwise the previous value of uncracked strains is replaced by the latest vector of uncracked strains, a new cracked strain vector is computed and the cracked flexibility matrix and the uncracked concrete stiffness matrix are calculated again. The iterative procedure is continued until the convergence requirement is satisfied.

The incremental stress for the cracked elements can then be calculated from the following constitutive relation for cracked reinforced concrete.

$$\{\Delta\sigma\}^2 = \left[\frac{\Lambda_c}{S_c}[F] + [D]^{-1}\right]^{-1}\{\Delta\epsilon\} \qquad 54$$

Thus, the total incremental stresses in the concrete attained during the current step are calculated from:

$$\{\Delta\sigma\} = \{\Delta\sigma\}^1 + \{\Delta\sigma\}^2 \qquad 55$$

Where:

$\{\Delta\sigma\}$ = total incremental stresses in concrete during current step

$\{\Delta\sigma\}^1$ = incremental stresses required to crack the element

$\{\Delta\sigma\}^2$ = incremental stresses for the cracked element

The total stresses sustained by the reinforced concrete element is obtained by adding the steel stresses to the concrete stresses computed from Equation 55.

It should be noted that the crack formation criteria used in the computer code is based on the maximum tensile stress criteria. A tensile crack is formed whenever the principal tensile stress exceeds the maximum tensile strength of the concrete. Once the crack is formed, the stress in the concrete normal to the crack is set to zero and the concrete strain at which the crack ocurred is stored. The crack is assumed to close whenever the concrete strain is smaller than the strain at which the crack ocurred. For closing cracks, the constitutive relations used are similar to those used for the initially uncracked concrete. The crack is assumed to open again whenever the concrete compressive stress drops to zero,whereupon for subsequent loading, the constitutive equations for cracked reinforced concrete, given by Equation 54, are used.

3. Computer Program for Nonlinear Analysis of Reinforced Concrete

3.1 Introduction:

The computer program developed to model the nonlinear behavior of reinforced concrete plane stress or plain strain elements based on the theoretical concepts established in Chapter 2 is given in Appendix A2. A flow chart is first discussed herein to establish the sequence of principal operations performed by the program, followed by a description of the activities performed within each subroutine included in the code.

The computer subroutines presented in Appendix A2 are to be incorporated in an existing computer program at the Air Force Weapons Laboratory identified by the acronym of SAMSON (6). This code is used to perform nonlinear dynamic analysis of plane and axisymmetric problems but at present considers only the nonlinear stress-strain relation for concrete. The material subroutine discussed subsequently should enhance the analytical capabilities of the computer code SAMSOM.

The main purpose of the computer code is to calculate the incremental stresses induced by a prescribed vector of incremental strains for an uncracked or cracked reinforced concrete element. The material subroutines require that the prescribed incremental strain vector be defined beforehand. The code does not include an equation solving subroutine as those operations will be performed by the principal code SAMSOM.

3.2 Sequential Operations of Computer Program:

The following operations are performed by the material subroutine code for each finite element in the analysis.

A. Read Material information and initialize variables for element considered.

B. Determine incremental strain vector for current step.

C. Compute set of constants required for the endochronic model.

D. Compute stress increment in concrete for prescribed incremental vector.

   1. Change stresses and strains from global to principal directions.

   2. Check for previous cracking in the element. If element has cracked proceed to step D7.

   3. Compute endochronic parameters and incremental stress vector for concrete.

4. Check if new cracks have formed. If the principal tensile stress is smaller than the concrete tensile strength go to step E.

5. Determine order and number of cracks.

6. Update stresses in current step to state of incipient cracking.

7. Start iteration for uncracked strains. Determine distribution of incremental strain vector left in current step between the uncracked and cracked concrete.

8. Determine total cracked directions.

9. Determine crack flexibility matrix.

10. Determine uncracked concrete stiffness matrix.

11. Compute new vector of uncracked strains.

12. If number of iterations for uncracked strains is smaller than three go to step D7.

13. Compute stresses in cracked concrete element.

E. Compute incremental stress vector for reinforcement caused by prescribed incremental strains.

F. Transform incremental stress vectors for concrete and reinforcement to global directions.

G. Update stresses and strains.

H. Proceed to next strain increment

I. End

3.3 Description of Code Subroutines:

The computer code presented in Appendix A2 contains 14 subroutines in addition to the main section of the program. The main section is used to compute the incremental strain vector according to the analysis desired and to read the control and material data required. The subroutines are described subsequently.

<u>Subroutine COEF</u>: This subroutine, called by the main section of the program, computes the constant coefficients required for the endochronic equations given in Appendix A1. Input required are the concrete compressive strength, the reinforcement yield stress and the reinforcement ratios in each direction. This subroutine does not call any other subroutine.

<u>Subroutine CRACHK</u>: This subroutine is called by subroutine MATER1 to determine if initial cracks have ocurred during the current stress increment or to determine if the cracks in a previously cracked element are closed or open. Input required are the stress and strain vectors for the current and past steps and the strain at which the crack opened previously. This subroutine does not call any other subroutine.

<u>Subroutine CRASTI</u>: This subroutine is called by subroutine ONECRA to compute the crack flexibility matrix. Required input are the previous strain and stress vector, the proportion of incremental strain contributed by the cracks, and the geometric properties of the element such as bar diameter, concrete cover, number of bars, etc. This subroutine does not call any other subroutine.

<u>Subroutine FUNEND</u>: This subroutine is called by subroutines MATER1 and ONECRA to compute the endochronic model. Subroutine MATERI calls FUNEND to compute the incremental stress vector for uncracked concrete caused by the prescribed incremental strains. Subroutine ONECRA calls FUNEND to compute the uncracked concrete stiffness matrix for the uncracked incremental strain vector and to compute the cracked concrete incremental stresses. Required input is the stress and strain vector for the current and previous step and whether the element is in plane strain, plane stress or axisymmetric. This subroutine calls subroutines INVAR and PRIN.

<u>Subroutine GLOBAL</u>: This subroutine is called by subroutine MATER1 to transform the computed concrete and reinforcement stress vectors in the

current step, referred to the principal axis, back to the global coordinates. Required input is the stress vector to be computed and the angle between the principal axis and the global coordinate axis. This subroutine does not call any other subroutine.

Subroutine INV: This subroutine is called by other subroutines to compute the inverse of a given matrix. Required input are the matrix to be inverted, the array where the results will be stored, and the order of the matrix. This subroutine does not call any other subroutine.

Subroutine INVAR: This subroutine is called by subroutine FUNEND to compute the stress and strain invariants for the current values of the strain and stress vectors. Input required are the current values of the strain and stress vectors, together with the incremental strain vector. Subroutine INVAR does not call any other subroutine.

Subroutine MATER1: This subroutine is called by the main section of the program to compute the incremental stress vector in the concrete and reinforcement caused by the prescribed strain vector. Input required for this subroutine is the current and previous step vectors of stress and strain, the total stress vector of steel stresses and whether the element is in plane stress, plain strain or axisymmetric. This subroutine calls the subroutines ROTATE, FUNEND, CRACHK, ONECRA, and STEEL.

Subroutine MATMU; This subroutine is called by other subroutines as required to multiply two given matrices. Input required is the name of the two matrices to be multiplied, the name of the array where the results will be stored, and the order of the matrices. This subroutine does not call any other subroutine.

Subroutine ONECRA: This subroutine is called by subroutine MATER1 when initial cracking in an element is detected. The subroutine updates the stresses to the state of incipient cracking, determines the proportion of total strain increment required to complete the current step strain increment, performs the iteration on the cracked and uncracked concrete strains, and computes the stresses in the cracked concrete. The input required are the current and previous step strain and stress vectors referred to principal and global directions, the strains at which cracking previously ocurred, the angle between the principal and global directions and the cracking direction. This subroutine calls subroutines CRASTI and FUNEND.

Subroutine PRIN: This subroutine is called by subroutines ROTATE and FUNEND. Subroutine ROTATE calls PRIN to determine the principal stresses or strains from the global stress or strain vector. Subroutine FUNEND calls PRIN to arrange the maximum, intermediate and minimum principal stress in increasing order of magnitude. Input required is the stress or strain vector and whether principal stresses are to be computed or not. This subroutine does not call any other subroutine.

Subroutine ROTATE: This subroutine, called by MATER1, computes the principal stresses and strains for the current and previous strain and stress vectors. Input required are the strain and stress vectors for the current and past steps. This subroutine calls subroutine PRIN.

Subroutine Steel: This subroutine is called by MATER1 to compute the incremental stress vector induced in the reinforcement by a prescribed strain increment. Required input are the current and previous strain vectors in principal and global coordinates. It should be noted that in this subroutine the bars are assumed to be oriented along the element global directions (See Figure 1). This subroutine does not call any other subroutine.

Subroutine ZER: This subroutine is called by the other subroutines as required to initialize the values of arrays to zero. Input required is the matrix to be initialized and its order. No other subroutine is called by ZER.

4. Numerical Results:

4.1 Introduction:

In this chapter, the results of several experimental programs are compared with the results predicted by the computer program in order to ascertain its applicability. Comparisons are made with experimental stress-strain curves for plain concrete in uniaxial and biaxial stress states, and for reinforced concrete panels subjected to biaxial stress states.

4.2 Stress strain Curve for Plain Concrete:

In this section, the stress strain curve for plain concrete predicted by the computer code is compared with experimental results for uniaxial and biaxial stress states. The prescribed strains are increased in 0.0001 increments and the resulting induced stresses are computed from the constitutive relation given in Equation 15 for the corresponding stress states.

In Figure 3, the computed stress strain curve for uniaxial loading of plain concrete is compared with the experimental data given in Reference 10 for various concrete strengths. The endochronic formulation coded in the program predicts curves that are very similar to the observed behavior.

Figure 4 compares the stress strain curve for biaxial stress states computed by the program with the experimental points determined from Reference 14. As for the uniaxial tests, the predicted values agree well with the observed behavior.

4.3 Stress-Strain Curves for Reinforced Concrete Panels Subjected to Biaxial Stress States:

Several experimental results have been reported in the literature (16, 18) where reinforced concrete panels have been subjected to monotonic and cyclic shear stresses. In this section the results of one test reported by

Perdikaris, et al (16), are used to compare the results predicted by the computer code. Additional comparisons with experimental data should be conducted when the code is incorporated to the main program to determine its general applicability to more complex problems.

The test specimen reported by Perdikaris, et al, had a central section of 24 in by 24 in with a thickness of 6 in and reinforced in one direction with one layer of #4 bars at 6" spacing and with two layers of #4 bars at 6" spacing in the perpendicular direction. The concrete compressive strength was 3160 psi, while the reinforcement yield stress was reported as 61000 psi. The specimen was subjected to a monotonic shear stress up to failure without the application of biaxial normal stresses. The ultimate shear stress sustained by the specimen was reported at 475 psi.

The experimental load deflection curve determined for this specimen is given in Figure 5 together with the calculated load displacement relation. In general, both curves corelate well but the model fails to predict the correct ultimate shear stress. It has been observed that the crack flexibility matrix becomes illconditioned at large strains and predicts significantly different results for the cracked incremental stresses. It is believed that the problem is caused by the equation proposed the flexibility coefficient F_3.

5. Conclusions:

The computer code developed herein can be used to model the nonlinear behavior of cracked reinforced concrete in finite element analysis. At this point, the code considers the nonlinear stress-strain curve of plain concrete and of the reinforcement, the anisotropy induced by complex stress states and by cracking, and the postcracking shear transfer mechanisms. The code can be used for the nonlinear analysis of reinforced concrete structures once it is incorporated into the main program SAMSOM.

The program predicts results that agree reasonably well with a limited amount of experimental data for plain and reinforced concrete elements. A large number of additional tests should be conducted when the material subroutine is incorporated in the main program to check the program predictions with the available experimental data on reinforced concrete panels reported by Perdikaris, et. al. (16) and by Vecchio, et. al (18). This effort will determine the final application range of the material subroutine developed herein.

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FIGURES

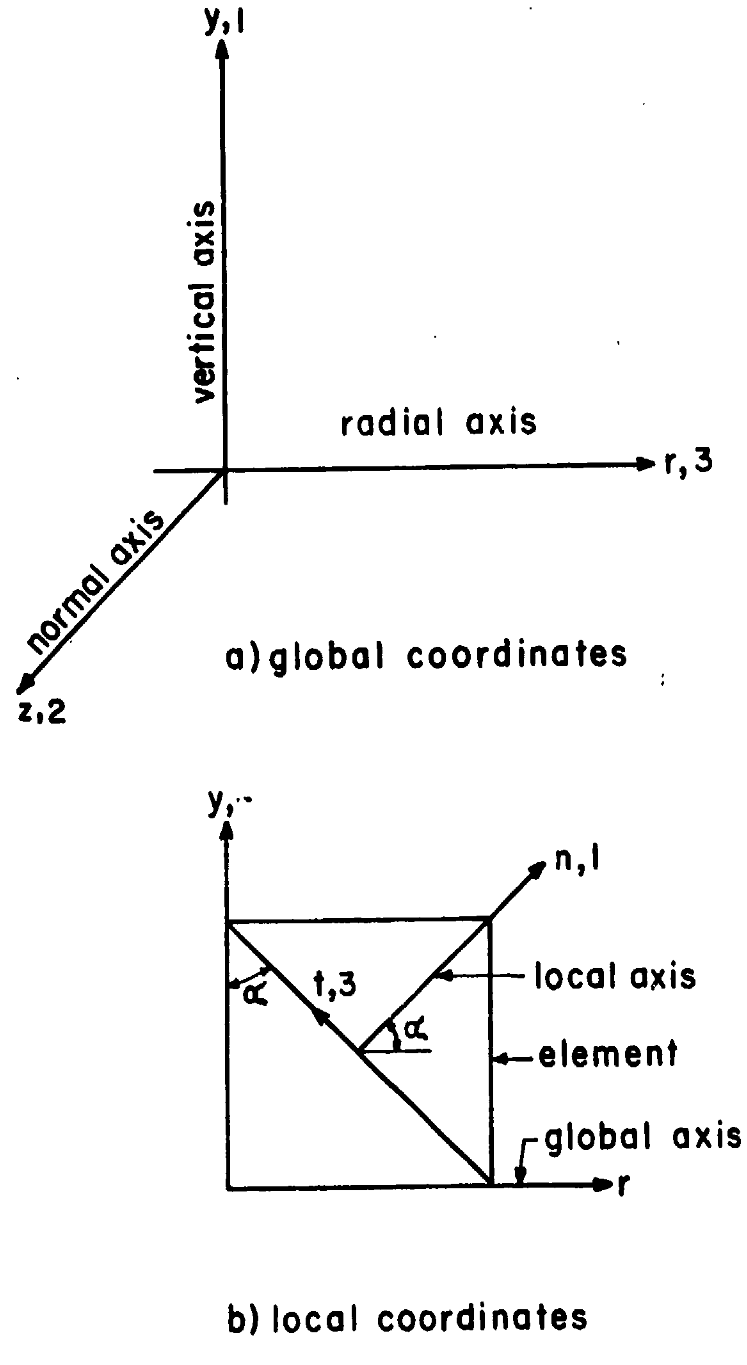

Figure 1: Coordinate Definition

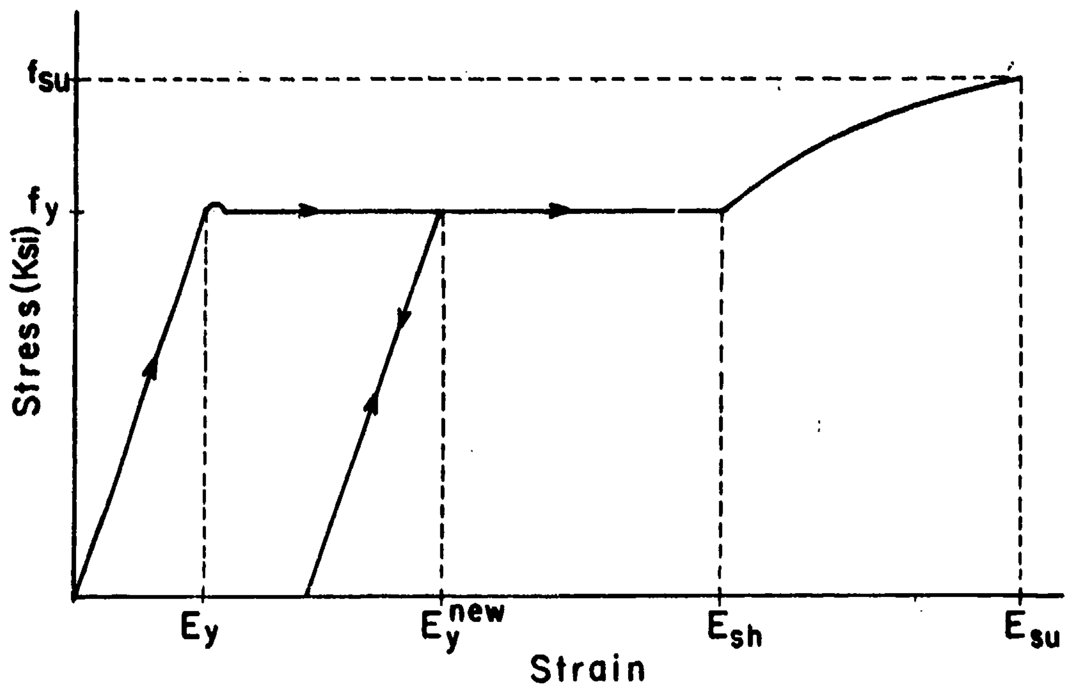

Figure 2: Typical stress-strain culve for Monotonic And Repeated Loading.

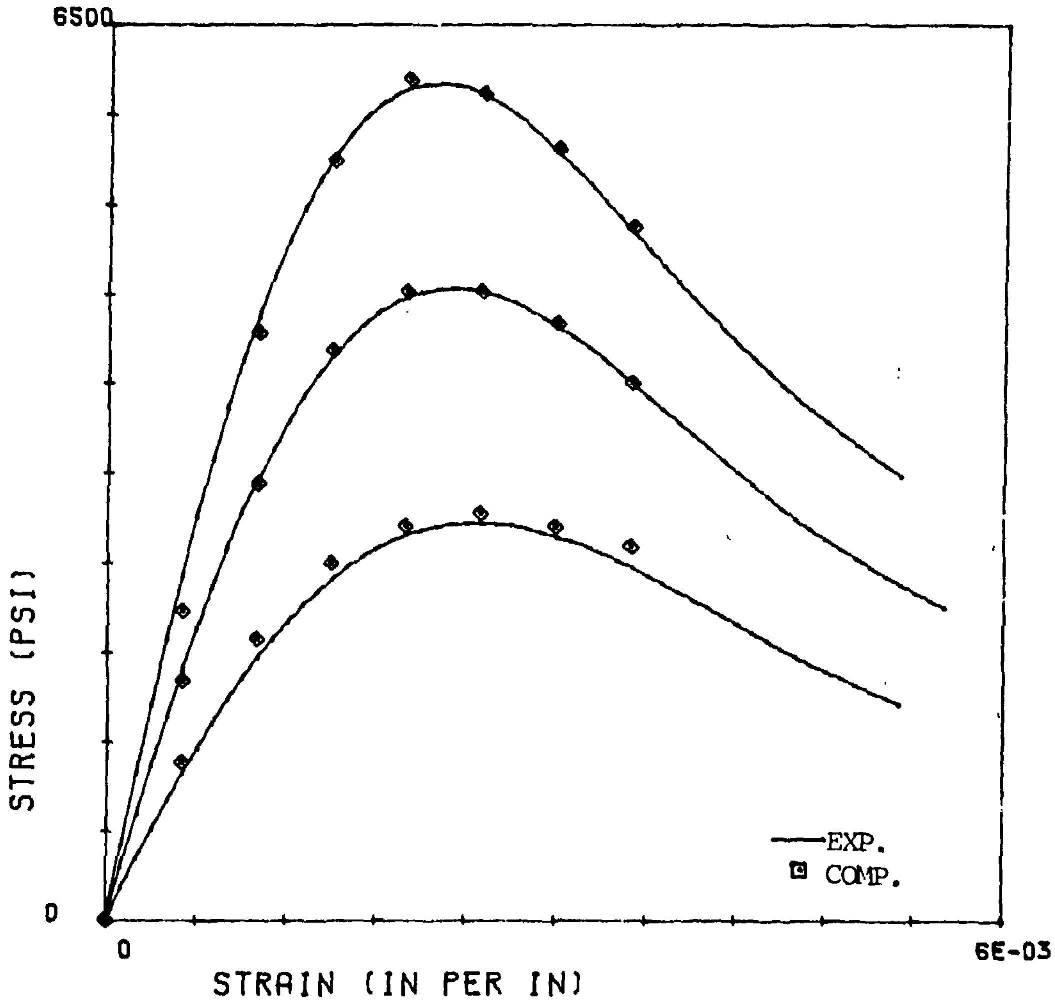

FIGURE #3: COMPUTED STRESS-STRAIN CURVE FOR UNIAXIAL LOADING OF PLAIN CONCRETE

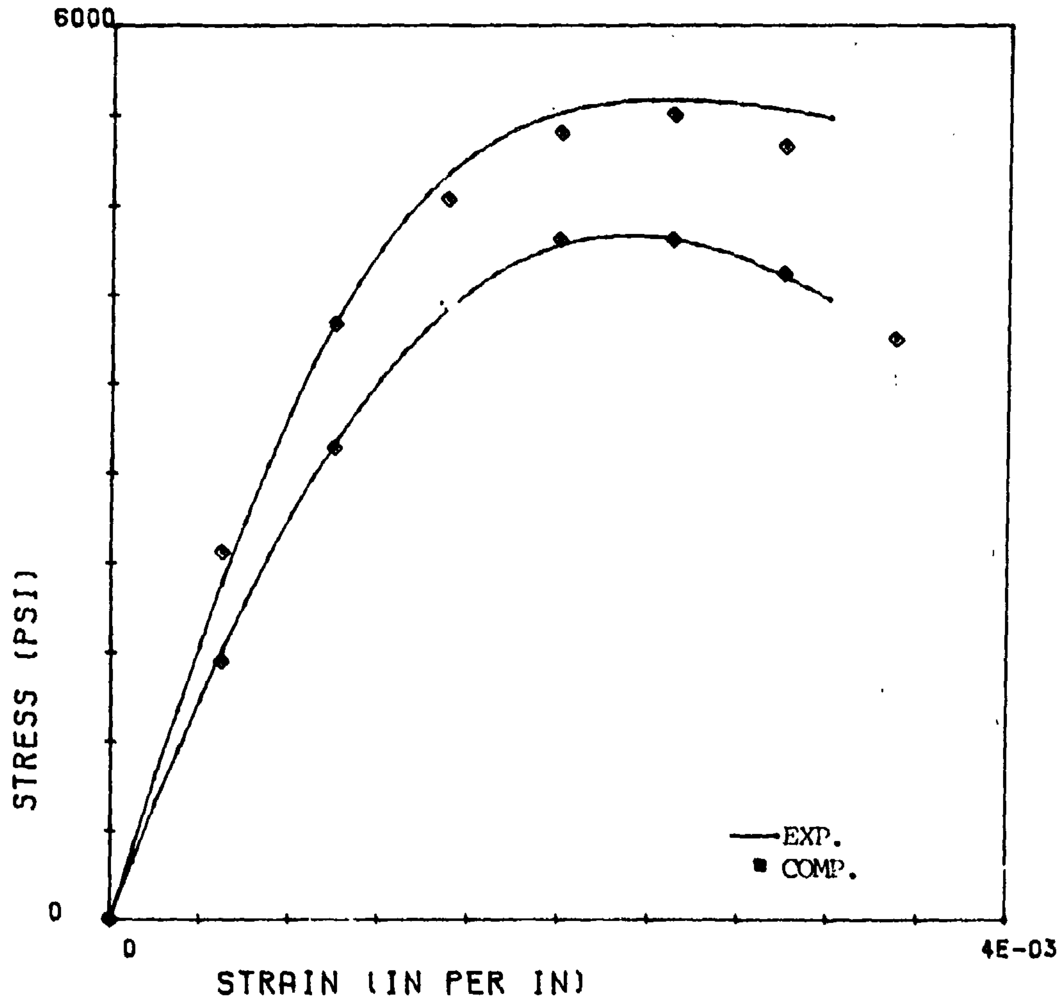

FIGURE 4: COMPUTED STRESS-STRAIN CURVE FOR BIAXIAL LOADING OF PLAIN CONCRETE

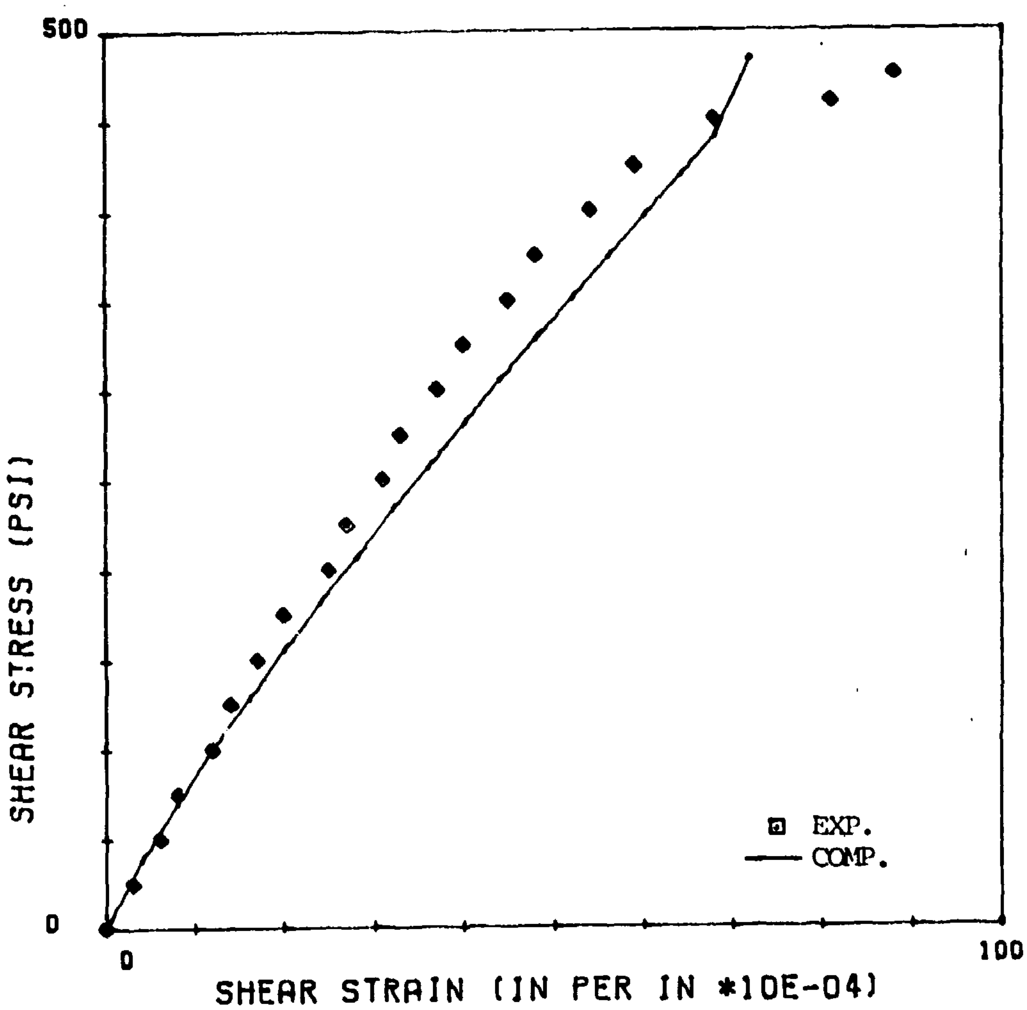

FIGURE 5: LOAD-DEFLECTION CURVE FOR REINFORCED CONCRETE PANEL SUBJECTED TO BIAXIAL STRESSES

APPENDIX 1

A. 1 Summary of Equations for Endochronic Model

The equations presented herein are summarized from Ref. 5 , to compute the material constants and functions required to model the nonlinear behavior of concrete.

Distortion Intrinsic Time Parameter Z:

$$\Delta Z = \frac{\Delta\zeta}{Z_1} \qquad \text{A1.1}$$

$$\Delta\zeta = \frac{\Delta N}{f} \qquad \text{A1.2}$$

$$\Delta\eta = (F)\ \Delta\xi \qquad \text{A1.3}$$

$$\Delta\xi = (J_2\ (\Delta\epsilon)\ )^{1/2} \qquad \text{A1.4}$$

$$F = F_1 + F_2 \qquad \text{A1.5}$$

$$F_1 = \frac{a_o\ (1-g_1)}{1 - a_5\left[I_3(\sigma)\right]^{1/3}\ (1 + g_2)} \qquad \text{A1.6}$$

$$F_2 = \frac{F_2'}{F_2''} \qquad \text{A1.7}$$

$$F_2' = a_2\ \sqrt{J_2(\epsilon)}\ \ (1+\left|a_6\ I_2(\sigma)\right|^{1/4} + F_5) \qquad \text{A1.8}$$

$$F_2'' = 1-a_1 I_1(\sigma) + \left|a_8\ I_2\ (\sigma)\right|^{1/4} F_4 - a_3 I_3(\sigma)\left[J_2(\sigma)\right]^{1/8}(1+g_2) \qquad \text{A1.9}$$

$$f = \left[1 + \frac{\beta_1\ \eta + \beta_2\ \eta^2}{1 + F_2/a_7}\right] F_3 \qquad \text{A1.10}$$

$$F_3 = 1 + \frac{a_{10}}{J_2(\epsilon)\ \ (1+a_9/\eta^2)} \qquad \text{A1.11}$$

$$g_1 = g_{11}\ g_{12} \qquad \text{A1.12}$$

$$g_2 = g_{21}\ g_{22}\ g_{23} \qquad \text{A1.13}$$

$$g_{11} = a_{14}\ \ J_2(\epsilon)^{1/4}\left(\frac{\sigma_{med} - \sigma_{min}}{\sigma_{max} - a_{23}}\right)\left[a_{15}\left(\frac{\sigma_{med} - \sigma_{min}}{\sigma_{max} - a_{23}}\right)^{4/3} - a_{16}\right] \qquad \text{A1.14}$$

$$g_{12} = 1 - \left[1 + \left|\frac{\sigma_{min}}{a_{17}(\sigma_{max} - a_{23})}\right|^4\right]^{-1} \tag{A1.15}$$

$$g_{21} = \left[\frac{g_{21}'}{g_{21}''}\right]^{5/4} \tag{A1.16}$$

$$g_{21}' = a_{18}\left[\frac{\sigma_{med} - \sigma_{min}}{\sigma_{med} - a_{23}}\right]^{-1} \tag{A1.17}$$

$$g_{21}'' = a_{19}\left[1 - a_{20}\frac{|\sigma_{min}|}{(\sigma_{max} - a_{23})}\right](\sigma_{min} - a_{23}) \tag{A1.18}$$

$$g_{22} = \left[1 + a_{21}\left[\frac{\sigma_{min}}{\sigma_{max} - a_{23}}\right]^4\right]^{-1} \tag{A1.19}$$

$$g_{23} = \left[\frac{\left[J_2(\varepsilon)\right]^{1/4}}{a_{22} + \left[J_2(\varepsilon)\right]^{1/2}}\right]^3 \tag{A1.20}$$

$$F_4 = \left[\frac{\left[J_2(\varepsilon)\right]^{1/4}}{a_4 + \left[J_2(\varepsilon)\right]^{1/2}}\right]^3 \tag{A1.21}$$

$$F_5 = a_{11}\sigma_{min}(1 + a_{12}\sigma_{min})\left[\frac{\left[J_2(\varepsilon)\right]^{1/4}}{|a_{13}\sigma_{min}|^{1/4} + \left[J_2(\varepsilon)\right]^{1/2}}\right]^3 \tag{A1.22}$$

<u>Compaction Intrinsic Time Parameter Z':</u>

$$\Delta Z' = \frac{\Delta\zeta'}{Z_2} \tag{A1.23}$$

$$\Delta\zeta' = \frac{\Delta\eta'}{h} \tag{A1.24}$$

$$\Delta\eta' = (H)\ \Delta\xi' \tag{A1.25}$$

$$\Delta\xi' = |I_1(\Delta\varepsilon)| \tag{A1.26}$$

$$h = 1 + \frac{\eta'}{\beta_3} + \left[\frac{\eta'}{\beta_4}\right]^2 \tag{A1.27}$$

$$H = b_1\left[\frac{J_1(\sigma)}{b_2 - I_1(\sigma)}\right]^2 \tag{A1.28}$$

Inelastic Dilatancy Parameter λ:

$$\Delta\lambda = (\ell)\ (L)\ (\Delta\xi) \quad \text{A1.29}$$

$$\ell = 1 - \frac{\lambda}{\lambda_o} \quad \text{A1.30}$$

$$L = \frac{c_3}{1-c_1 I_1(\sigma)} \left[\left[\frac{\lambda}{\lambda_o} \right]^2 + \left[\frac{c_4 J_2(\varepsilon)}{c_2^{\,2} + J_2(\varepsilon)} \right]^2 \right] \quad \text{A1.31}$$

Shear Compaction Parameter λ':

$$\Delta\lambda' = (\ell')\ (L') \quad \text{A1.32}$$

$$\ell' = c_6\ (1- \lambda' / \lambda_o') \quad \text{A1.33}$$

$$L' \quad \frac{\sigma_{min}\, g_3^{\ 1/3}}{1 + | g_3 / c_8 |^3} \quad \text{A1.34}$$

$$g_3 = 0.93\ | c_7\ \sigma_{min} | - \sqrt{J_2(\varepsilon)} \quad \text{A1.35}$$

Bulk and Shear Modulus:

$$K = \frac{1}{1 + c_5 \lambda} \quad \frac{E_o}{3(1-2\nu)} \quad \text{A1.36}$$

$$G = \frac{1}{1 + c_5\lambda} \quad \frac{E_o}{2(1+\nu)} \quad \text{A1.37}$$

Stress and Strain Invariants:

The following equations are valid for principal stresses and strains only:

$$I_1(\sigma) = \sigma_{11} + \sigma_{22} + \sigma_{33} \quad \text{A1.38}$$

$$I_2(\sigma) = \frac{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2}{6} \quad \text{A1.39}$$

$$I_3(\sigma) = (\sigma_{11})\ (\sigma_{22})(\sigma_{33}) \quad \text{A1.40}$$

$$I_1(\Delta\varepsilon) = \Delta\varepsilon_{11} + \Delta\varepsilon_{22} + \Delta\varepsilon_{33} \quad \text{A1.41}$$

$$J_2(\Delta\varepsilon) = \frac{(\Delta\varepsilon_{11} - \Delta_{22})^2 + (\Delta\varepsilon_{22}-\Delta\varepsilon_{33})^2 + (\Delta\varepsilon_{33} - \Delta\varepsilon_{11})^2}{6} \quad \text{A1.42}$$

$$J_2(\varepsilon) = \frac{(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2}{6} \quad \text{A1.43}$$

Material Constants:

$$a_o = 0.7 \qquad \text{A1.44}$$

$$a_1 = 0.6/f_c' \qquad \text{A1.45}$$

$$a_2 = 1400 \left[\frac{f_c'}{4650}\right]^{1/2} \qquad \text{A1.46}$$

$$a_3 = \frac{324000}{(f_c')^4} \qquad \text{A1.47}$$

$$a_4 = 0.045 \qquad \text{A1.48}$$

$$a_5 = \frac{2160}{(f_c')^2} \qquad \text{A1.49}$$

$$a_6 = \frac{0.15}{(f_c')^2} \qquad \text{A1.50}$$

$$a_7 = 0.05 \qquad \text{A1.51}$$

$$a_8 = \frac{15}{(f_c')^2} \left[\frac{f_c'}{3600}\right]^{1.5} \qquad \text{A1.52}$$

$$a_9 = 1.5 \times 10^{-3} \qquad \text{A1.53}$$

$$a_{10} = 1.25 \times 10^{-4} \qquad \text{A1.54}$$

$$a_{11} = \frac{0.2}{f_c'} \qquad \text{A1.55}$$

$$a_{12} = \frac{0.8}{f_c'} \qquad \text{A1.56}$$

$$a_{13} = \frac{2.2 \times 10^{-5}}{f_c'} \qquad \text{A1.57}$$

$$a_{14} = 25 \qquad \text{A1.58}$$

$$a_{15} = 1.095 \qquad \text{A1.59}$$

$$a_{16} = 1.216 \qquad \text{A1.60}$$

$$a_{17} = 0.055 \qquad \text{A1.61}$$

$$a_{18} = 0.94 \qquad \text{A1.62}$$

$$a_{19} = \frac{6300}{(f_c')^2} \qquad \text{A1.63}$$

$$a_{20} = 14 \qquad \text{A1.64}$$

$$a_{21} = 1000 \tag{A1.65}$$

$$a_{22} = 0.04 \tag{A1.66}$$

$$a_{23} = 0.2\ (f_c') \tag{A1.67}$$

$$b_1 = 9.1\left(\frac{f_c'}{7020}\right) \tag{A1.68}$$

$$b_2 = f_c' \tag{A1.69}$$

$$c_1 = \frac{2}{f_c'} \tag{A1.70}$$

$$c_2 = 3 \times 10^{-3} \tag{A1.71}$$

$$c_3 = 0.5 \tag{A1.72}$$

$$c_4 = 2.0 \tag{A1.73}$$

$$c_5 = 150 \tag{A1.74}$$

$$\beta_1 = 30 \tag{A1.75}$$

$$\beta_2 = 3500 \tag{A1.76}$$

$$\beta_3 = 0.08 \tag{A1.77}$$

$$\beta_4 = 0.23 \tag{A1.78}$$

$$Z_1 = 0.0015 \tag{A1.79}$$

$$Z_2 = 0.0125 \tag{A1.80}$$

$$\lambda_o = 0.003 \tag{A1.81}$$

$$\nu = 0.18 \tag{A1.82}$$

$$c_5 = 0.002 \tag{A1.83}$$

$$c_7 = 1.05 \times 10^{-6} \tag{A1.84}$$

$$c_8 = 0.001 \tag{A1.85}$$

$$\lambda_o' = 0.003 \tag{A1.86}$$

$$E_o = 4 \times 10^6 + (f_c' - 4650) \times 10^3 \tag{A1.87}$$

The concrete strength parameter f_c' used to compute the material constants should be given in psi.

APPENDIX A2: Computer Program Listing

```
      DIMENSION SIGI(6),SIGI1(10,6),EPSTI(6),EPSTI1(10,6)
         DIMENSION DER(4),DUM(4)
C     COM C(60),D(1,6)
      DATA ISTRES,IESLAST,IPLAST,WGT/2,0,0,0
      DATA E1(1),E2(1),E3(1),E(4),E5(1)/0,0,0,0,0
      DO 117 I=1,4
C
C  EPSTI1(I,J)=PREVIOUS STRAIN OF ELEMENT I IN DIRECTION J
C  EPSTI(I)=CURRENT STRAIN IN DIRECTION I
C  SIGI1(I,J)=PREVIOUS STRESS OF ELEMENT I IN DIRECTION J
C  SIGI(I)=CURRENT STRESS IN DIRECTION I
C  TST(I)=TOTAL STEEL STRESS IN DIRECTION I
C
         EPSTI1(1,I)=0.0
      EPSTI(I)=0
      SIGI1(1,I)=0
      SIGI(I)=0
      TST(I)=0
117   CONTINUE
      STINC=0.0001
C
C INITIALIZE OR READ PREVIOUS STRAINS AND STRESSES
C
      EPSTI(4)=EPSTI(4)+STINC
      IONE=1
C  FC=CONCRETE COMPRESSIVE STRENGTH
C  FY=REINFORCEMENT YIELD STRESS
C  ECT=MAXIMUM TENSILE STRAIN IN CONCRETE
C  PS1,PS2,PS3 IS REINFORCEMENT RATIO IN DIRECTIONS 1,2 AND
3
      DATA FC,FY,ECT,PS1,PS2,PS3/2263.,34809.,-.01,.0179,0.,
.0131
      CALL COEF(FC,FY,ECT,PS1,PS2,PS3)
C
C
C
C
230   CALL MATER1(SIGI,SIGI1,EPSTI,EPSTI1,TST,C,DCR,DUN,ISTR
ES)
      DO 118 I=1,4
      SIGI1(1,I)=SIGI(I)
      EPSTI1(1,I)=EPSTI(I)
118   CONTINUE
      EPSTI(4)=EPSTI(4)+STINC
       GO TO 230
      END
      SUBROUTINE MATER1(SIG,SIG1,EPS,EPS1,TST,C,DCR,DUN,ISTR
ES)
      DIMENSION ST(6),THE(4),DS(4,4),CRASTR(4),DEUN(4),DECR(
4)
      I=1
      FT=C(53)
      ITE=0
      CRADIR=0
      NTCR=0
       ITE=0
C
C SAVE STRESSES AND STRAINS BEFORE CHANGING TO PRINCIPAL DIR
ECTIONS
C
      DO 119 I=1,4
      S1(I)=SIG1(1,I)
      S(I)=SIG(I)
      E1(I)=EPS1(1,I)
      E(I)=EPS(I)
119   CONTINUE
*
```

```
C
C CHANGE STRESSES AND STRAINS TO PRINCIPAL DIRECTIONS
C
      CALL ROTATE(S,S1,E,E1,THE)
        IF(THE(2).EQ.0.0.AND.THE(4).NE.0.0)THE(2)=THE(4)
      DO 120 I=1,3
      IF(CRASTR(I).EQ.0.0) GO TO 120
      NCR=NCR+1
      ICR=I
120     CONTINUE
        IF(NCR.EQ.0)GOTO2000
      IF(NCR.EQ.1.AND.ICR.EQ.2)GOTO2000
      IF(NCR.EQ.0)GOTO2000
      GOTO 5000
C COMPUTE ENDOCHRONIC PARAMETERS AND INCREMENTAL STRESSES
2000  CALL FUNEND(S,S1,E,E1,THE,DS,ISTRES,NCR)
C     CHECK FOR CRACKS IN CONCRETE (MAX. SRESS CRITERIA)
      CALL CRACHK(S,S1,E,E1,CRASTR,FT,NTCR,CRADIR)
      IF(NTCR.EQ.0)GOTO 8000
        IF(NTCR=1.AND.CRADIR.EQ.2)GOTO 2400
        IF(NTCR.EQ.1.AND.CRADIR.LT.4)GOTO2500
        IF(NTCR.EQ.2.AND.CRADIR.EQ.4)GOTO 4000
        IF(NTCR.EQ.3)GOTO 4000
        GOTO 2500
2400      S(2)=0
        IF(ISTRES.EQ.2)GOTO 8000
      GOTO 8000
C EXIT IF NEW CRACK IS IN 2(THETE) DIRECTION
2500      WRITE(3,27)
27    FORMAT(5X,'***WARNING:ELEMENT HAS CRACKED***')
 C CRACK IN  ONE DIRECTION/DETERMINE CRACKING DIRECTION
      ICR=CRADIR
      CALL ONECRA(S,S1,E,E1,SIG,SIG1,EPS,EPS1,CRASTR,C,TST,T
HE,DUN,DCR,A,FT,ICR,ISTRES)
        GOTO 8000
8000      CALL STEEL(ST,EPS,EPS1,E,E1,THE,STRMAX)
C CHANGE STEEL AND CONCRETE STRESSES TO GLOBAL COORDINATES
        B=2
      CALL GLOBAL(ST,A,B)
      DO 121 I=1,4
      SX(I)=S(I)
        EX(I)=E(I)
121   CONTINUE
      CALL GLOBAL(SX,A,B)
        B=1
        CALL GLOBAL(EX,A,B)
      DO 122 I=1,4
        EPS(I)=EX(I)
        E(I)=EPS(I)
           S(I)=SIG(I)=SX(I)
122   CONTINUE
      EPS(2)=E(2)
      DO 123 I=1,4
      TST(I)=TST(I)+ST(I)
      SU1=SIG(I)+TST(I)
      WRITE(3,28)SIG(I),TST(I),SU1
28      FORMAT(5X,'SCON=',E10.4,5X,'STEEL=',E10.4,5X,'TOSTR='
,E10.4)
123     CONTINUE
        STOP
        RETURN
*
```

```
      SUBROUTINE INVAR(S,E,DE,SIN1,SIN2,SIN3,DSIN2,DEIN1,DDE
IN2
C     OPTION BASE 1
C     THIS SUBROUTINE COMPUTES STRESS AND STRAIN INVARIANTS
C     STRESS INVARIANTS
      SAV=S(1)+S(2)+S(3)
      SAV=SAV/3.
      SIN1=SAV*3.
      SIN2=-(S(1)*S(2)+S(2)*S(3)+S(1)*S(3))
      SIN3=S(1)*S(2)*S(3)
      DSIN2=((S(1)-S(2))**2.+(S(2)-S(3))**2.+(S(3)-S(1))**2.
)/6.
C     STRAIN INVARIENTS
      DEAV=DE(1)+DE(2)+DE(3)
      DEAV=DEAV/3.
      DEIN1=DEAV*3.
      DDEIN2=((DE(1)-DE(2))**2.+(DE(2)-DE(3))**2.+(DE(3)-DE(
1))**2.)/6.
      DEIN2=((E(1)-E(2))**2.+(E(2)-E(3))**2.+(E(3)-E(1))**2.
)/6.
      WRITE(3,175)SIN1,SIN2,SIN3,DSIN2,DEIN1,DDEIN2,DEIN2
175    FORMAT(7G)
      RETURN
      END
      SUBROUTINE(FC,FY,ECT,PS1,PS2,PS3)
C     COM C(60),D(1,6)
      C(1)=0.7
      C(2)=0.6/FC
      C(3)=1400.*(FC/4650.)**0.5
      C(4)=90.*3600./FC**4.
      C(5)=0.045
      C(6)=0.6*3600/FC**2.
      C(7)=0.15/FC**2.
        C(8)=0.05
      C(9)=15./FC**2.*(FC/3600)**1.5
      C(10)=1.5E-3
      C(11)=1.25E-4
      C(12)=0.2/FC
      C(13)=0.8/FC
      C(14)=2.2E-5/FC
      C(15)=25.
      C(16)=1.095
      C(17)=1.216
      C(18)=0.055
      C(19)=0.94
      C(20)=6300./FC**2.
      C(21)=14.
      C(22)=1000.
      C(23)=0.04
      C(24)=0.2*FC
      C(25)=9.1*FC/7020.
      C(26)=FC
      C(27)=2./FC
      C(28)=3.E-3
      C(29)=0.5
      C(30)=2
      C(31)=150.
      C(32)=30.
      C(33)=3500.
      C(34)=0.08
      C(35)=0.23
      C(36)=1.5E-3
      C(37)=0.0125
      C(38)=3E-3
*
```

```
        C(39)=0.18
       C(40)=0.002
       C(41)=1.05E-6
       C(42)=0.001
       C(43)=3E-3
       C(44)=4.E6+(FC-4650)*1000.
       C(45)=FY
       C(46)=29000000.
       C(47)=C(45)/C(46)
       C(48)=8.*C(47)
       C(49)=80.*C(47)
       C(50)=FS1
       C(51)=FS2
       C(53)=FS3
       C(53)=5.*SQR(C(26))
C   SC=CRACK SPACING
C   AC=AREA OF SHEAR PLANE
C   CM=MINIMUN CONCRETE COVER TO BAR
C   NBL=NUMBER OF BAR PER LAYER
C   NB=NUMBER OF LAYER IN SECTION
C   BN=NET BEAM WIDTH
       C(54)=4.
       DO 345 I=1,6
       D(1,I)=0.0
345    CONTINUE
       RETURN
       END
       SUBROUTINE ROTATE(S,S1,E,E1,THE)
       ZERO=0.0
       DO 176 I=1,4
       THE(I)=ZERO
176    CONTINUE
       ZERO=1
C      PREVIOUS STRESSES TO PRINCIPAL STRESSES
       CAL PRIN(S1,THE,ZERO)
C      CURRENT STRESSES TO PRINCIPAL STRESSES
       CALL PRIN(S,THE,ZERO)
   C   PREVIOUS STRAINS TO PRINCIPAL STRAINS
       E1(4)=E1(4)/2.
       ZERO =1.
       CALL PRIN(E1,THE,ZERO)
C      CURRENT STRAUNS TO PRINCIPAL STRAINS
       E(4)=E(4)/2.
       CALL PRIN(E,THE,ZERO)
       RETURN
       END
       SUBROUTINE FUNEND(S,S1,E,E1,THE,DC,ISTRES)
C      COM C(60),D(1,6)
       TOLETA=0.01
       TOLAM=TOLETA
       ZERO=0.0
       DLAMP=ZERO
       DLAM=DLAMP
       DETA=ZERO
       DETAP=ZERO
       ITE=ZERO
       DO 177 I=1,4
       DS(I)=ZERO
177    DE(I)=E(I)-E1(I)
11523  ITE=ITE+1
*
```

```
        DO 178 I=1,4
        SI(I)=SI(I)+DS(I)
        EI(I)=EI(I)+DE(I)
178      CONTINUE
        THE(5)=SI(4)
        XLAMI=D(1,1)+DLAM
        ETAI=D(1,3)+DETA
        XLAMPI=D(1,2)+DLAMP
        ETAPI=D(1,4)+DETAP
        CALL INVAR(SI,EI,DE,SIN1,SIN2,SIN3,DSIN2,DEIN1,DDEIN2,
DEIN2,ISTRES)
        ZERO=2.
        DO 181 I=1,4
181     STRESS(I)=SI(I)
        CALL PRIN(STRESS,THE,ZERO)
        SMAX=STRESS(3)
        SMIN=STRESS(1)
        SMED=STRESS(2)
        IF(ABS(SMAX).LT.1E-4)SMAX=0.0
        IF(ABS(SMED).LT.1E-4)SMED=0.0
        IF(ABS(SMIN).LT.1E-4)SMIN=0.0
        G3=ABS(C(41*SMIN)*.93-SQRT(DEIN2)
        IF (G3.GT.0.0)GO TO 11615
        XLPRI=SMIN*(ABS(G3)**(1/3)/(1+ABS(G3/C(42))**3.))
        GO TO 11620
11615 XLPRI=SMIN*(G3**(1/3)/(1+ABS(G3/C(42))**3.))
11620 SLPRI=C(40)*(1.-ABS(XLAMPI)/C(43))
        C1=1./(1.+C(31)*XLAMPI)
        EO=C(44)
        XK=C1*EO/(3.*(1.-2.*C(39)))
        G=C1*EO/(2.*(1.+C(39)))
        X1=(C(30)*DEIN2/(C(28)**2.+DEIN2))**2.
        X1=X1+(XLAMI/C(38))**2.
        X1=X1*(C(29)/(1.-C(27)*SIN1))
        S1=1.-XLAMI/C(33)
        H=C(25)*(SIN1/(C(26)-SIN1))**2.
        SH=1.+ETAPI/C(34)+(ETAPI/C(35))**2.
        F5=C(12)*SMIN*(1.+C(13)*SMIN)
        F5=F5*(DEIN2**0.25/(ABS(C(14)*SMIN)**0.25+DEIN2**0.5))
**3.
        F4=(DEIN2**0.25/(C(5)+DEIN2**0.25))**3.
        G23=(DEIN2**0.25/(C(23)+DEIN2**0.5))**3.
        G22=1+C(22)*(SMIN/(SMAX-C(24)))**4.
        G22=1/G22
        G21TOP=C(19)*(SMED-SMIN)/(SMED-C(24))-1.
        G21BOT=C(20)*(1.-C(21)*ABS(SMIN)/(SMAX-C(24)))*(SMIN-C
(24))
        G21=(G21TOP/G21BOT)**1.25
        D1=(SMED-SMIN)/(SMAX-C(24))
        D2=C(16)*D1**(4./3.)-C(17)
        G11=C(15)*DEIN2**0.25*D1*D2
        G12=1.+(SMIN/(C(18)*(SMAX-C(24))))**4.
        G12=1./G12
        G12=1.-G12
        G1=G11*G12
        G2=G21*G22*G23
        F2TOP=C(3)*SQRT(DEIN2)*(1.+ABS(C(7)*SIN2)**0.25+F5)
        F2BOT=1.-C(2)*SIN1+ABS(C(9)*SIN2)**0.25*F4-C(4)*SIN3*D
SIN2**.125
      **(1.+G2)
        F2=F2TOP/F2BOT
        IF(SIN3.GT.0.0)11850
        F1=C(1)*(1.+G1)/(1.+C(6)*ABS(SIN3)**(1./3.)*(1.+G2))
        GO TO 11860
*
```

```
11850 F1=C(1)*(1.-G1)/(1.-C(6)*ABS(SIN3)**(1./3.)*(1.+G2))
11860 F=F1+F2
      IF(ETAI.GT.0.0)GO TO 11870
      F3=1.
      GOTO 11880
11870 F3=1.+C(11)/(DEIN2*(1.+C(10)/ETAI**2.))
11880 SF=(1.+(C(32)*ETAI+C(33)*ETAI**2.)/(1.+F2/C(8)))*F3
C CALCULATE INCREMENTS IN LAMBDA,ETA,AND INTRINSIC TIME
      DPSI=SQRT(DDEIN2)
      DPSIP=SQRT(DEIN1**2.)
      DETA=F*DPSI
      DETAP=H*DPSIP
      DCHI=DETA/SF
      DCHIP=DETAP/SH
      DZ=DCHI/C(36)
      DZP=DCHIP/C(37)
      DLAMP=S1*X1*DPSI
      DELAMP=SLPRI*XLPRI*DPSI
C CALCULATE DEVIATORIC AND VOLUMETRIC STRESSES
      VSTRAI=DE(1)+DE(2)+DE(3)
      VSTRES=SI(1)+SI(2)+SI(3)
      IF(ABS(VSTRES).LT.1.E-12)VSTRES=0.0
      IF(ABS(VSTRAI)).LT.1.E-12)VSTRAI=0.0
      DO 191 I=1,3
      DSTRAI(I)=DE(I)-VSTRAI/3.
      DSTRES(I)=SI(I)-VSTRES/3.
      DSTRAI(I)=DE(4)
      DSTRES(4)=SI(4)
      DO 192 I=1,4
      B(I)=DSTRAI(I)/SQRT(4.*DDEIN2)
      XKO(I)=DSTRES(I)*F/(SF*C(36))+3.XK*(S1*X1+SLPRI*XLPRI)
192   IF(I.EQ.4.)XKO(I)=DSTRES(I)*F/(SF*C(36))
      BNN=B(1)+B(2)+B(3)
   C  MAT DC=ZER(4,4)
      DO 271 I=1,3
271   DC(I,I)=XK+4.*G/3.-VSTRES*H/(3.*SH*C(37))-XKO(I)*B(I)-
BNN/3.)
      DO 272 I=1,3
      K=I+1
      DO 273 J=K,3
      DC(I,J)=XK-2.*G/3.-VSTRES*H/(3.*SH*C(37))-XKO(I)*(B(J)
-BNN/3.)
      DC(J,I)=XK-2.*G/3.-VSTRES*H/(3.*SH*C(37))-XKO(J)*(B(I)
-BNN/3.)
273   CONTINUE
272   CONTINUE
      DO 274 J=1,3
      DC(4,J)=-XKO(4)*(B(J)-BNN/3.)
      DC(J,4)=-XKO(J)*B(4)
274   CONTINUE
      DC(4,4)=2.*G-XKO(4)*B(4)
C     SOLVE FOR STRESSES (1-PL STRAIN;2-PL STRESS)
      IF(ISTRES=0)GO TO 12410
      GO TO (12360,12410),ISTRES
12360 DO 275 I=1,4
      DSTRES(I)=0.0
      DO 276 J=1,4
      DSTRES(I)=DSTRES(I)+DC(I,J)*DE(J)
276   CONTINUE
275   CONTINUE
      GOTO 12500
12410 DO 278 I=1,4
      DSTRES(I)=0.0
*
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      IF(I=2)GO TO 12470
      DO 280 J=1,4
      DSTRESS(I)=DSTREE(I)+(DC(I,J)-DC(I,2)*DC(2,J)/DC(2,2))
*DE(J)
280   CONTINUE
12470 CONTINUE
      DE(2)=-(DC(2,1)*DE(1)+DC(2,3)*DE(3)+DC(2,4)*DE(4))/DC(
2,2)
C
C
C
      DO 12550 I=1,6
      DS(I)=DSTRESS(I)
12550 CONTINUE
I
      IF(ITE.LT.I) GO TO 12614
      CDLAM=ABS((DLAM-DLAM1)/DLAM)
      CDETA=ABS((DETA-DETA1)/DETA)
      IF(CDLAM.LT.TOLAM).AND.(CDETA.LT.TOLETA) GO TO 12630
      IF(ITE.GT.7) GO TO 12630
12614 DLAM1=DLAM
      DETA1=DETA
      GO TO 11523
C
C
C
C
C
C
C
C
      IF(ISTRES.EQ.0) GO TO 13500
      D(1,1)=D(1,1)+DLAM
      D(1,2)=D(1,2)+DLAMP
      D(1,3)=D(1,3)+DETA
      D(1,4)=D(1,4)+DETAP
C
C                   CONVERT STRESSES/STRAINS TO GLOBAL COORDI
NATES
C
        IF(NCR.GT.0)GOTO13500
      DO 12790 I=1,4
      E(I)=E1(I)+DE(I)
      S1(I)=S(I)
      S(I)=S(I)+DS(I)
12790 CONTINUE
13500 RETURN
      SUBROUTINE PRIN(STRESS,THE,ZERO)
      COMMON C(60)
      IF(ZERO.EQ.2.) GO TO 14120
      THE=STRESS(1)-STRESS(3)
      IF(THE.NE.0.) GO TO 14060
      IF(THE.EQ.0.AND.STRESS(4).GT.0.) THE=PI/4.
      IF(THE.EQ.0.AND.STRESS(4)LT.0) THE=3.*PI/4.
      GO TO 14080
14060 THE=2.*STRESS(4)/THE
      THE=ATN(THE)/2.
14080 STEMP=(STRESS(1)+STRESS(3))/2.
      TAUMAX=SQRT(((STRESS(1)-STREE
      TAUMAX=SQRT(((STRESS(1)-STRESS(3))/2.)**2.+STRESS(4)**
2.)
      STRESS(4)=0.
      STRESS(1)=STEMP+TAUMAX
      STRESS(3)=STEMP-TAUMAX
*
```

```
      IF(STRESS(1).LT.F1).AND.(STRESS(3).LT.F1) GO TO 14120
14120 SMIN=MAX1(STRESS(1),STRESS(2),STRESS(3))
      SMAX=MIN1(STRESS(1),STRESS(2),STRESS(3))
      IF (ZERO.EQ.1.) RETURN
      II=III
      II=0
      III=0
      DO 14290 I=1,3
      IF (STRESS(I).EQ.SMAX) GO TO 14200
      IF (STRESS(I).EQ.SMIN) GO TO 14250
      SMED=STRESS(I)
       GO TO 14320
14200 II=II+1
      IF (II.EQ.I) IMED=I
      IF (II.EQ.2) GO TO 14290
      IMAX=I
      GO TO 14290
14250 III=III+1
      IF (III.EQ.2) IMED=I
      IF (III.EQ.2) GO TO 14290
      IMIN=I
14290 CONTINUE
      IF(III.EQ.3).OR.(II.EQ.3)
      IF (III.NE.3.AND.II.NE.3) GO TO 14310
      IMAX=I
      IMED=I
      IMIN=I
14310 SMED=STRESS(IMED)
14320 STRESS(1)=SMIN
      STRESS(2)=SMED
      STRESS(3)=SMAX
14390 RETURN
      SUBROUTINE STEEL(ST,EPS,EPS1,E,E1,THE,STRMAX)
      DIMENSION DS(4,4)
      COMMON C(60),D(1,6)
C     THIS SUBROUTINE IS CALLED BY MATER1.
C
C      COMPUTE STEEL STIFFNESS FOR REPEATED LOADING.
C      DETERMINE STEEL MODULUS IN STEEL LOCAL DIRECTIONS (R/
Z)
      DO 14660 J=1,3
      I=J
      IF((EPS(I)*EPS1(1,I)).LT.0.) GO TO 14520
      DES(J)=ABS(EPS(I))-ABS(EPS1(1,I))
      GO TO 14530
14520 DES(J)=EPS(I)-EPS(1,I)
14530 IF (DES(J).GT.0.) GO TO 14560
      ES(J)=C(46)
      GO TO 14660
C       STRAIN HARDENING
      ES(J)=10000000.
      STRMAX=EPS(I)
14660 CONTINUE
C
C
C
      DO 14700
      CALL ZER(DS,4,4)
      A=THE(2)
        DS(1,1)=C(50)*ES(1)*COS(A)**4.+C(52)*ES(3)*SIN(A)**4
*
      DS(3,1)=(SIN(A)*COS(A))**2.*(C(50)*ES(1)+C(52)*ES(3))
*
```

```
      DS(4,1)=2.*(SIN(A)*COS(A))*(C(50)*ES(1)*COS(A)**2.-C(5
2)*ES(3)*
     1SIN(A)**2.)
      DS(2,2)=C(51)*ES(2)
      DS(3,3)=C(50)*ES(1)*SIN(A)**4.+C(52)*ES(3)*COS(A)**4.
      DS(4,4)=(2.*COS(A)*SIN(A))**2.*(C(50)*ES(1)+C(52)*ES(3
))
      DS(3,4)=2.*SIN(A)*COS(A)*(C(50)*ES(1)*SIN(A)**2.-C(52)
*ES(3)
     1*COS(A)**2.)
      DS(3,1)=DS(1,3)
        DS(1,4)=DS(4,1)
      DS(4,3)=DS(3,4)
      DO 14830 I=1,4
      DE(I)=E(I)-E1(I)
14830 CONTINUE
      DO 14880 I=1,4
      ST(I)=0.
      DO 14870 J=1,4
      ST(I)=ST(I)+DS(I,J)*DE(J)
14870 CONTINUE
14880 CONTINUE
15500 RETURN
      SUBROUTINE GLOBAL(SX,A,B)
      DIMENSION DS(4,4),SO(4)
      CALL ZER(DS,4,4)
      DS(1,1)=COS(A)**2.
      DS(3,3)=COS(A)**2.
      DS(1,3)=SIN(A)**2.
      DS(3,1)=DS(1,3)
      DS(1,4)=-B*COS(A)*SIN(A)
      DS(2,2)=1.
      DS(3,4)=-DS(1,4)
      DS(4,1)=COS(A)*SIN(A)
        IF(B.EQ.1.)DS(4,1)=2.*COS(A)*SIN(A)
      DS(4,3)=-DS(4,1)
      DS(4,4)=COS(A)**2.-SIN(A)**2.
      DO 15660 I=1,4
      SO(I)=0.
      DO 15660 J=1,4
      SO(I)=SO(I)+DS(I,J)*SX(J)
15660 CONTINUE
      DO 15700 I=1,4
      SX(I)=SO(I)
15700 CONTINUE
      RETURN
      SUBROUTINE CRACHK(S,S1,E,E1,CASTR,FT,NTCR,CRADIR)
C
C
C    CHECK FOR CRACKS IN THREE DIRECTIONS
N
      DO 16080 I=1,3
      IF(CASTR(I).EQ.0.).AND.(S(I).LT.FT) GO TO 15830
      IF (S(I).GT.FT).AND.(CASTR(I).EQ.0.) GO TO 15850
      IF (CASTR(I).NE.0.) GO TO 15900
15830 CONTINUE
C    NO CRACKING IN THIS DIRECTION
      GO TO 16080
15850 CONTINUE
C     INITIAL CRACKING OCCURS (TENSION OR COMPRESION STRAIN
FIELD)
      PRO=(FT-S1(I))/(S(I)-S1(I))
      CASTR(I)=E1(I)+PRO*(S(I)-S1(I))
      CRADIR=I
*
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```
      NTCR=NTCR+1
      GO TO 16080
15900 CONTINUE
N       ELEMENT PREVIOUSLY CRACKED IN THIS DIRECTION
      IF (E1(I).GT.CASTR(I)).AND.(E(I).GT.CASTR(I)) GO TO 15
950
      IF (E1(I).GT.CASTR(I)).AND.(E(I).LT.CASTR(I)) GO TO 15
980
      IF (E1(I).LT.CASTR(I)).AND.(E(I).LT.CASTR(I)) GO TO 16
070
      IF (S1(I).LT.0.AND.S(I).GT.0.) GO TO 16020
15950 CONTINUE
C    CRACK REMAINS OPEN
      NTCR=NTCR+1
      GO TO 16080
15980 CONTINUE
C     OPEN CRACK CLOSES
      NTCR=NTCR+1
      CASTR(I)=E(I)
      GO TO 16080
16020 CONTINUE
C     CLOSED CRACK OPENS
      PRO=-S1(I)/(S(I)-S1(I))
      CASTR(2)
      CASTR(I)=E1(I)+PRO*(E(I)-E1(I))
      NTCR=NTCR+1
      GO TO 16080
16070 CONTINUE
C     CLOSED CRACK REMAINS CLOSED
16080 CONTINUE
16090 RETURN
      SUBROUTINE ONECRA(S,S1,E,E1,SIG,SID1,EPS,EPS1,
     *CASTR,C,TST,THE,DUN,DCR,A,FT,ICR,ISTRES)
      DIMENSION ST(4),ET(4),DECR(4),DEUN(J),DE(4),F(4,4),SXX
(4),DC(4,4)
     1,FD(4,4),THE(5)
        SC=C(54)
        AC=55.
      I=ICR
C
      ITER=0
       IF(FT.EQ.0)GOTO16510
      PRO=(FT-S1(I))/(S(I)-S1(I))
C      UPDATE STRESSES TO INCIPIENT CRACKING
      DO 16420 I=1,4
      SXX(I)=EPS(I)
      ST(I)=S1(I)+PRO*(S(I)-S1(I))
      S1(I)=ST(I)
      E1(I)=E1(I)+PRO*(E(I)-E1(I))
      EPS(I)=EPS1(1,I)+PRO*(EPS(I)-EPS1(1,I))
16420 CONTINUE
        B=2.
      CALL GLOBAL(ST,A,B)
      DO 16500 I=1,4
      SIG(I)=ST(I)
      SID1(1,I)=SIG(I)
        IF(I.NE.ICR)GO TO 16480
      S(I)=S1(I)
        S1(I)=0
16480      EPS1(1,I)=EPS(I)
      EPS(I)=SXX(I)
        IF(I.EQ.2)EPS(2)=E1(2)
16500 CONTINUE
*$
```

```
C       DETERMINE PROPORTION OF TOTAL STRAIN INCREMENT
        DO 16560 I=1,4
        DEUN(I)=(E(I)-E1(I))/2.
        DECR(I)=DEUN(I)
        DE(I)=E(I)-E1(I)
          IF(I.NE.2)GO TO 16560
          DEUN(2)=DE(2)
          DECR(2)=0
16560   CONTINUE
C       DETERMINE TOTAL CRACKED DIRECTIONS
        NCR=0
        DO 16620 I=1,3
        IF(CASTR(I).EQ.0.OR.I.EQ.2) GO TO 16620
        NCR=NCR+1
16620   CONTINUE
16640   CONTINUE
        CALL ZER(F,4,4)
        CALL ZER(DC,4,4)
        ITER =ITER+1
        CALL CRASTI(S1,DCR,DECR,C,F,TST,THE,A,NCR)
        DUM=0.
        DO 16700 I=1,4
        ET(I)=DUN(I)+DEUN(I)
          ET1(I)=DUN(I)
16700   CONTINUE
        CALL FUNEND(S,S1,ET,ET1,THE,DC,DUM,NCR)
C       COMPUTE UNCRACKED STRAINS
        CALL ZER(FD,4,4)
        CALL MATMU(F,DC,FD,4, 1,4)
        DO 16800 I=1,4
          FD(I,I)=FD(I,I)+1
16800   CONTINUE
        CALL INV(FD,DC,4)
        DO 16880 I=1,4
        DEUN(I)=0.
        DO 16860 J=1,4
        DEUN(I)=DEUN(I)+DC(I,J)*DE(J)
16860   CONTINUE
        DECR(I)=DE(I)-DEUN(I)
16880   CONTINUE
         IF (ITER.GT.2) GO TO 16920
        GO TO 16650
16920   CONTINUE
C       COMPUTE STRESSES IN CRACKED AND UNCRACKED CONCRETE
          ISTRES=DUM1
        DO 16960 I=1,4
        ET(I)=DUN(I)+DEUN(I)
          ET1(I)=DUN(I)
16960   CONTINUE
        CALL FUNEND(S,S1,ET,ET1,THE,DC,ISTRES,NCR)
          CALL CRASTI(S1,DCR,DECR,C,F,TST,THE,A,NCR)
          CALL ZER(FD,4)
          CALL INV(FD,DC,4)
          CALL ZER(SC,4)
          DO 1000 I=1,4
          DO 1000 J=1,4
          SC(I,J)=F(I,J)+FD(I,J)
          NEXT J
          NEXT I
          CALL  ZER(FD,4)
          CALL INV(FD,SC,4)
*
```

```
      DO 17060 I=1,4
      SCON(I)=0.
      DO 17050 J=1,4
      SCON(I)=SCON(I)+FD(I,J)*DE(J)
17050 CONTINUE
        S(I)=S1(I)+SCON(I)
        E(I)=E1(I)+DE(I)
        DCR(I)=DCR(I)+DECR(I)
        DUN(I)=DUN(I)+DEUN(I)
17060 CONTINUE
       RETURN
      SUBROUTINE CRASTI(S1,E1,DECR,C,F,TST,THE,A,NCR)
      ITE=0
C      COMPUTE FLEXIBILITY COEFFICIENTS FOR CRACKED PLANE
      SC=C(54)
      DB=C(56)
       AC=C(55)
      CM=C(5)
      NB1=C(58)
       NB=C(59)
        BN=C(60)
      FC=C(26)
      FY=C(45)
       VO=1.0
        FS=0.0
        CW=ABS(DCR(1)+DECR(1))*SC
       IF(ITE.EQ.1) CW=ABS(DCR(3)+DECR(3))*SC
18170 CD=ABS(DECR(4))*SC
      XKN==590000*C(50)
      XKN=590000*C(50)
      IF (ITE.EQ.1) XKN=590000*C(52)
      DUM=(3.4E5-XKN/CW)*1.09E-7
      IF (DUM.GT.0.) GO TO 18220
      DUM=0.
18220 CO=CW
      IF (CO.LT.5E-3) CO=.003
      AST=1000*AC/(3.9*(CO-.002)+DUM)
      VDY=.92*DB**2.*SQRT(FC+FY/1E6)
      VDO=DB*BN/NB1*(.47+.54*CM/(BN/NB1**2.+DB))
      VDU=VDY
      IF (VDO.LT.VDY) VDU=VDO
      A1=2.
      A2=0.
      FS=TST(1)
      IF (ITE.EQ.1) FS=TST(3)
      FC=FC/1000
      B1=1.-2.*FS/FY
      IF (B1.LT.O.) B1=0.
      B2=FS*DB*SQRT(FC)/(.003*A1*FY)
      DSTOP=-(A2*VDU/A1-2.*B2*CD))2.*B2
      DSTOP=-(A2*VDU/A1-2.*B2*CD)*2.*B2+2.*B1*DB*VDU*SQRT(FC
)/(.003*A1)
      IF (CD.NE.0.) GO TO 18350
      DST=1000.*312.*NB1*DB**1.75*NB
      GO TO 18380
18350 DSBOT=(A2*VDU/A1-2.*B2*CD)**2.+4.*B1*DB*SQRT(FC)*VDU*C
D/(.003*A1)
      DSBOT=2.*SQRT(DSBOT)
      DST=B2+DSTOP/DSBOT
      DST=DST*1000.*NB
      F4=AST+DST
      F4=1./F4
*
```

```
        C=ABS(CW)
        IF(C.LT.0.01)C=0.01
      F3=(3.9/XKN+1.09E-7*XKN/(CW*(XKN*CW+1.)))*VO/1000.
      AL1=AST/(AST+DST)
      F2TOP=.176*CW**(-.63)+(.22*CW**(-.552)-1.034)*FC
      F2BOT=.1353*CW**(-.8)+(.164*CW**(-.707)-1.379)*FC
      F2=F2TOP*AL1/(F3BOT*XKN*1000.)
      F4=1./(XKN*1000.)
      CALL ZER(F,4,4)
      IF (ITE.EQ.1) GO TO 20150
      F(1,1)=F1*AC/SC
      F(1,4)=F2*AC/SC
      F(4,1)=F3*AC/SC
      F(4,4)=F4*AC/SC
      IF (NCR.EQ.1) RETURN
       DB=C(56)
       AC=C(55)
      CM=2.625
      NB1=1
      NB=4
      BN=5.25
      FS=0.
      ITE=ITE+1
      GO TO 18170
20150 F(3,3)=F1*AC/SC
      F(3,4)=-F2*AC/SC
        F(4,3)=-F3*AC/SC
        F(4,4)=F(4,4)+F4*AC/SC
      RETURN
      SUBROUTINE ZER(W,N,M)
      DIMENSION W(N,M)
      DO 1 I=1,N
      DO 1 J=1,M
1     W(I,J)=0.0
      RETURN
      END
      SUBROUTINE MATMU(A,B,C,N,M,L)
      DIMENSION A(N,M),B(M,L),C(N,L)
      DO 1 I=1,N
      DO 1 J=1,L
      C(I,J)=0.0
      DO 1 K=1,M
1     C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
*
```